Institute of Media, Information, and Network

Chapter 1 Signals and Systems

Instructor: Hongkai Xiong (熊红凯)
Distinguished Professor (特聘教授)
http://min.sjtu.edu.cn

TAs: Yuhui Xu, Qi Wang

Department of Electronic Engineering
Shanghai Jiao Tong University

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About Lecturers

- Hongkai Xiong, Distinguished Professor (1-8 weeks)

  - Office: 1-309, No.1, SEIEE Bld.
  - Email: **xionghongkai@sjtu.edu.cn**
  - Web-page: **http://min.sjtu.edu.cn**

- Chenglin Li, Associate Professor (9-16 weeks)

  - Office: 1-304, No.1, SEIEE Bld.
  - Email: **lcl1985@sjtu.edu.cn**
  - Web-page: **http://min.sjtu.edu.cn/lcl/Chenglin_Li.htm**

About TA

- Yuhui Xu, Ph.D candidate
- Email: yuhuixu1993@126.com

- Qi Wang, Ph.D candidate
- Email: wang_qi@sjtu.edu.cn
- Office: Room 304, No.1, SEIEE Bld.

About The Class

- Requirements and Grading:

  - Homework & Project + Mid-term Test: 50%

  - Final Exam : 50%

About The Class

- Text book and reference:
  - Signals & Systems (Second Edition)
    by Alan V. Oppenheim, 电子工业出版社

References

《信号与系统》刘树棠，西安交通大学出版社

《复变函数》严镇军，中国科学技术大学出版社

《信号与系统》（上、下）郑君理，高等教育出版社

Topic

- 1.0 INTRODUCTION
- 1.1 CONTINUOUS-TIME AND DISCRETE-TIME SIGNALS
- 1.2 TRASFORMATION OF INDEPENDENT VARIABLE
- 1.3 EXPONENTIAL AND SINUSOIDAL SIGNALS
- 1.4 THE UNIT IMPULSE AND UNIT STEP FUNCTIONS
- 1.5 Definitions and Representations of Systems
- 1.6 BASIC SYSTEM PROPERTIES

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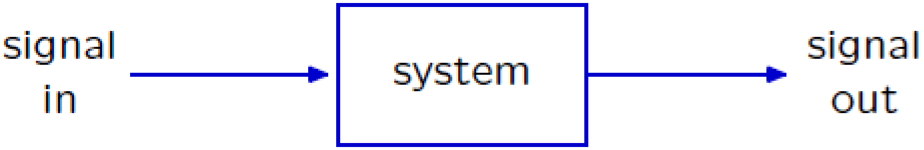
Introduction

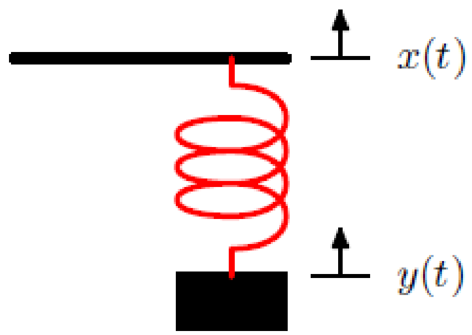
- Signals
  - Definitions, representations and classifications
  - Fundamental signal transformations
  - Typical signal examples
- Systems
  - Concepts, representations, and classifications
  - Basic properties of systems

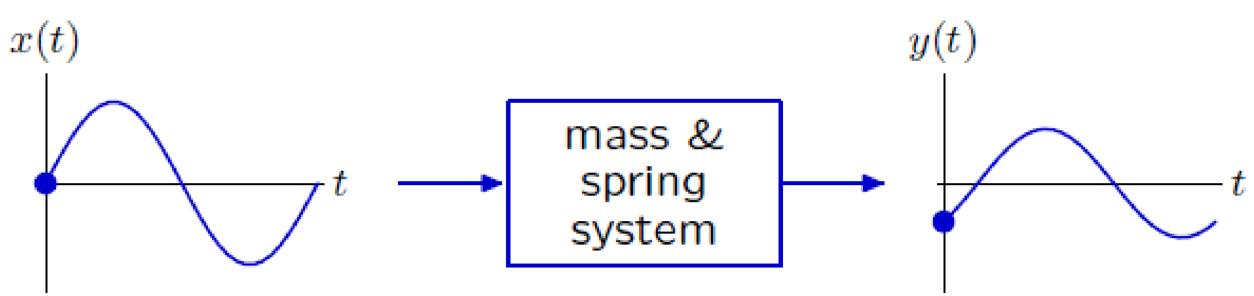
The Signals and Systems Abstraction

- Describe a system (physical, mathematical, or computational) by the way it transforms an input signal into an output signal.

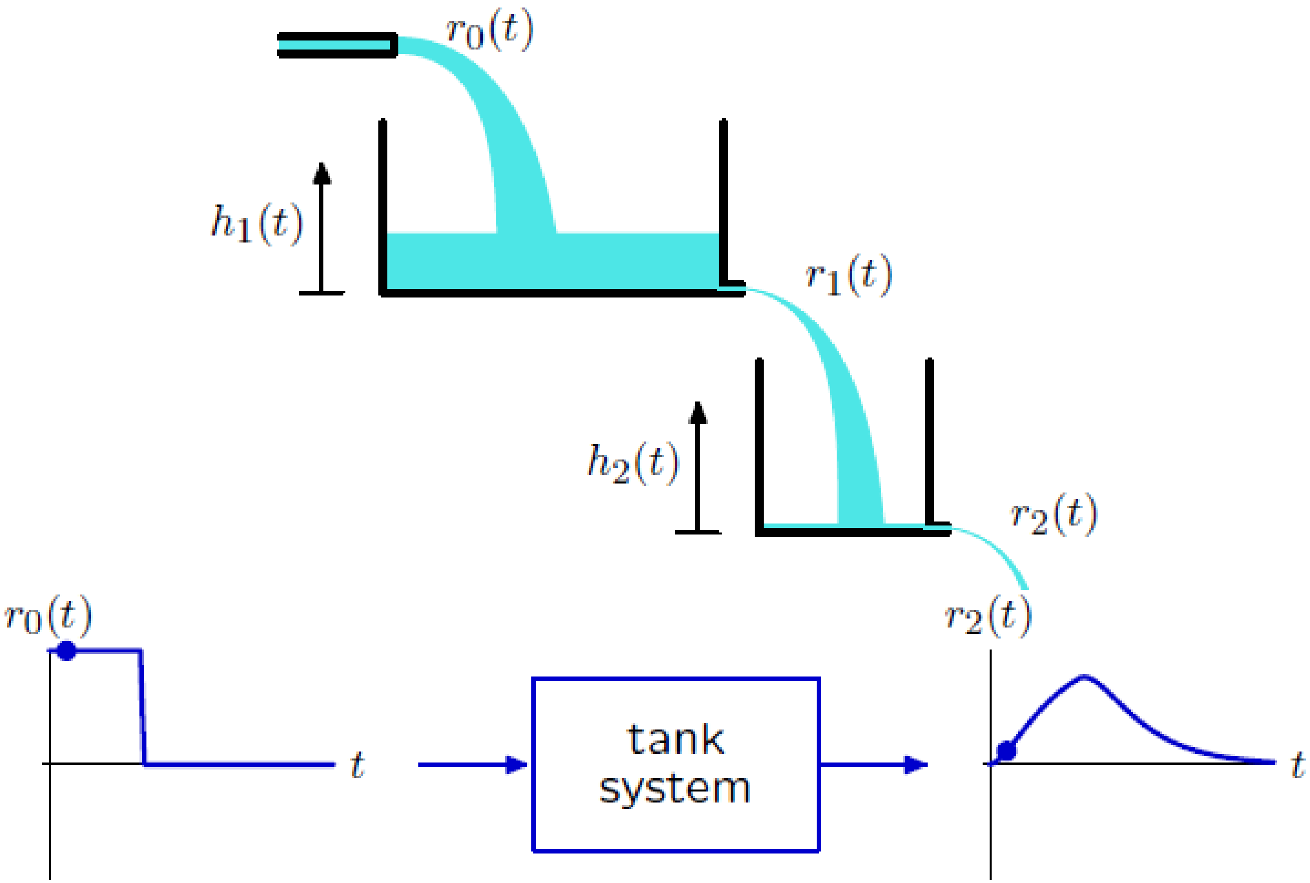


Example: Mass and Spring

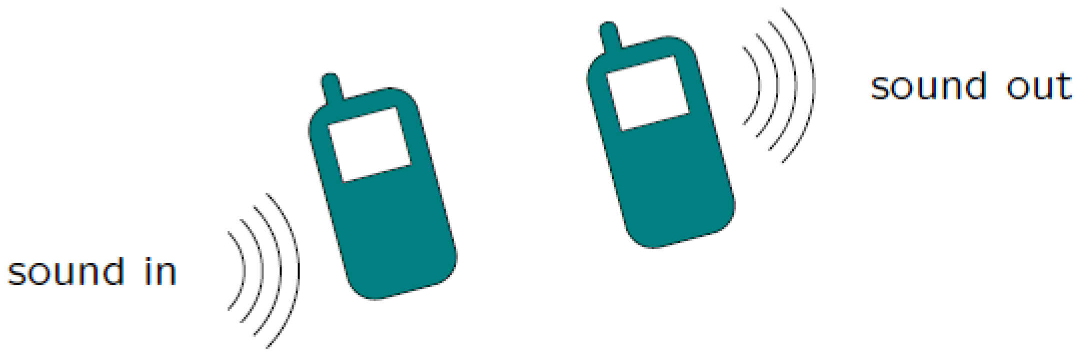


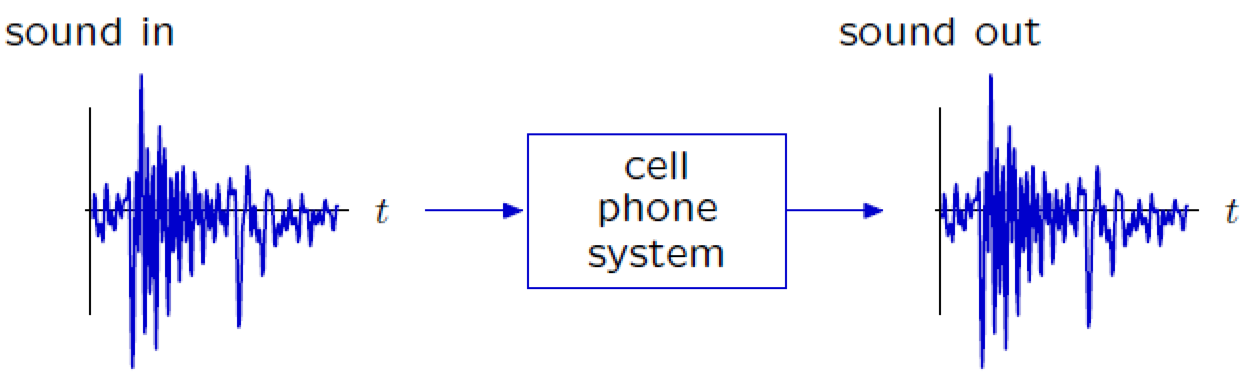


Example: Tanks



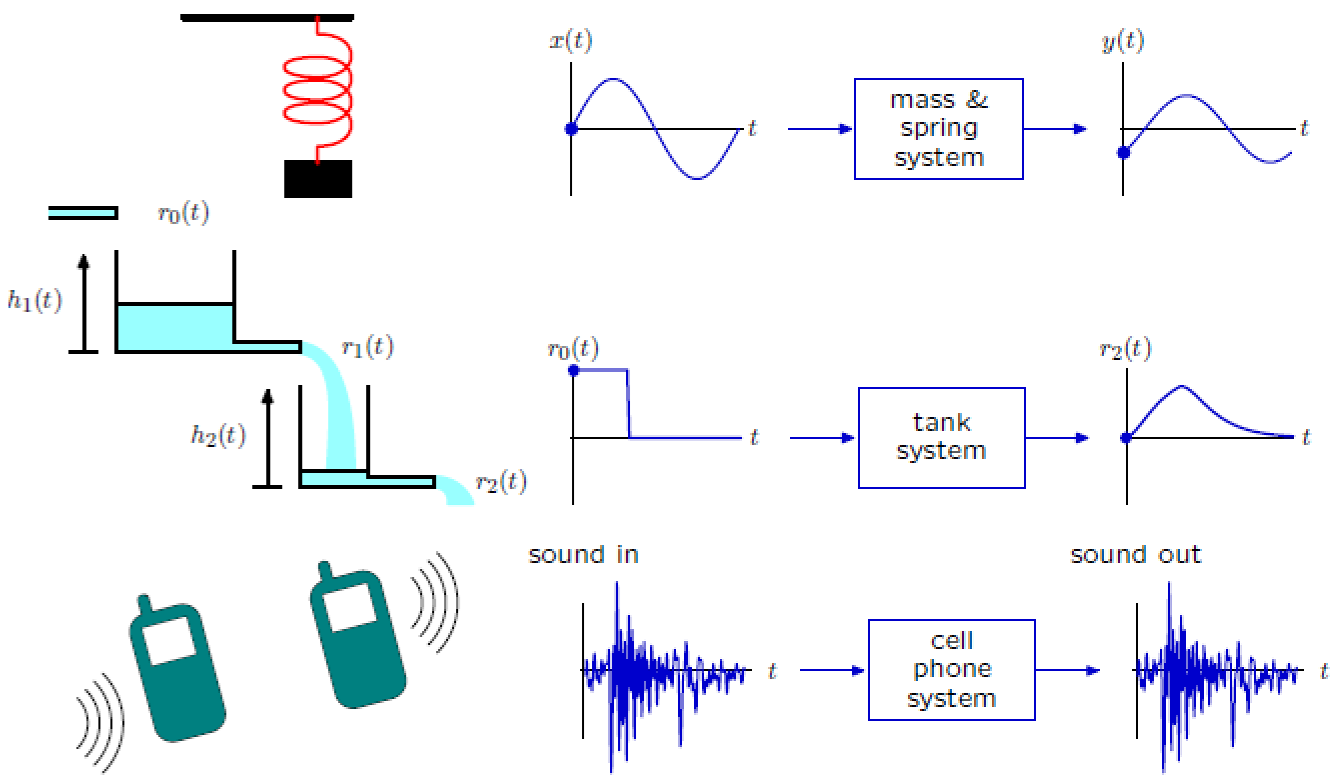
Example: Cell Phone System





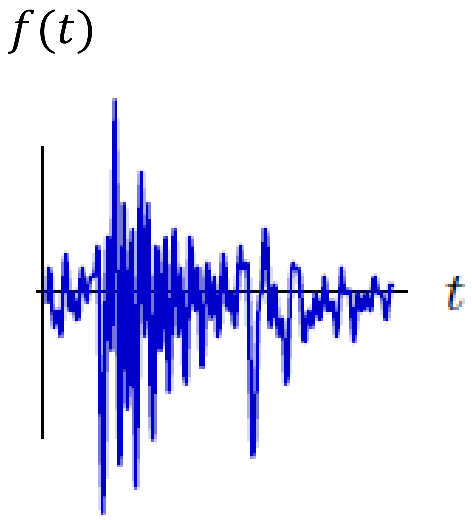
Signals and Systems: Widely Applicable

- The Signals and Systems approach has broad application: electrical, mechanical, optical, acoustic, biological, financial, ...



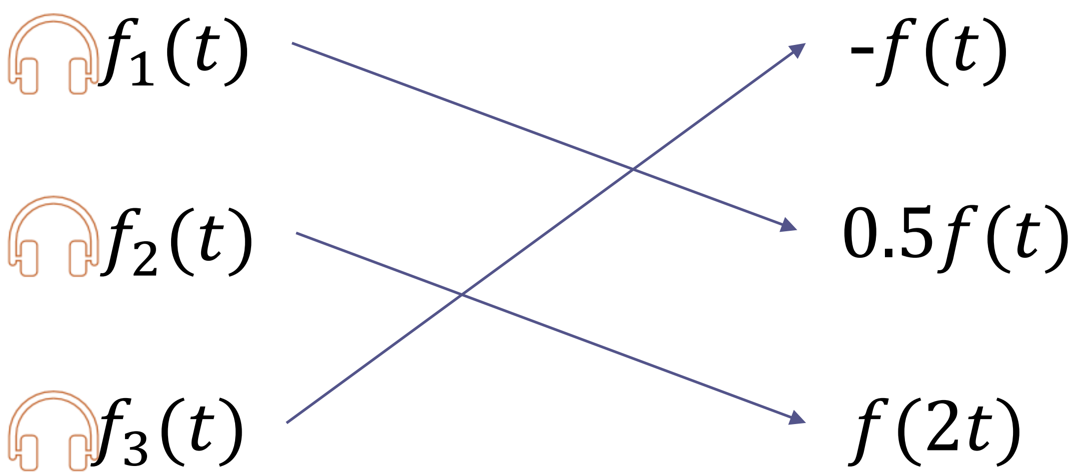
Check Yourself

- Computer generated music $f(t)$

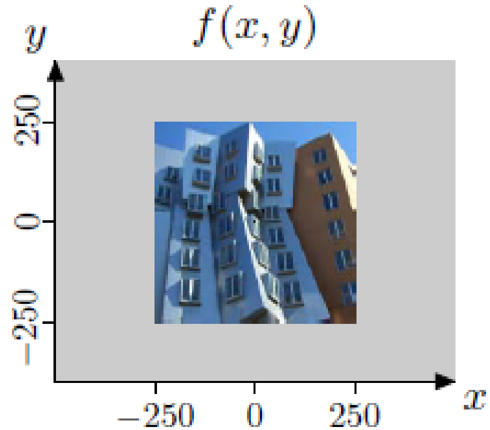


Check Yourself

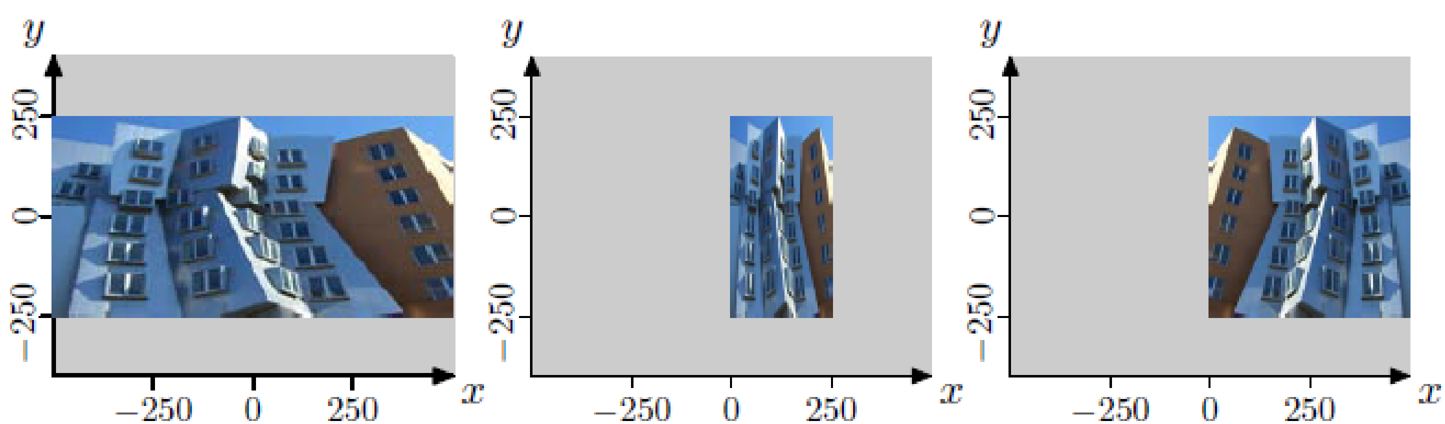
- Listen to the following three manipulated signals: $f_1(t)$ $f_2(t)$ $f_3(t)$, try to find the correct answer



Check Yourself



How many images match the expressions beneath them?

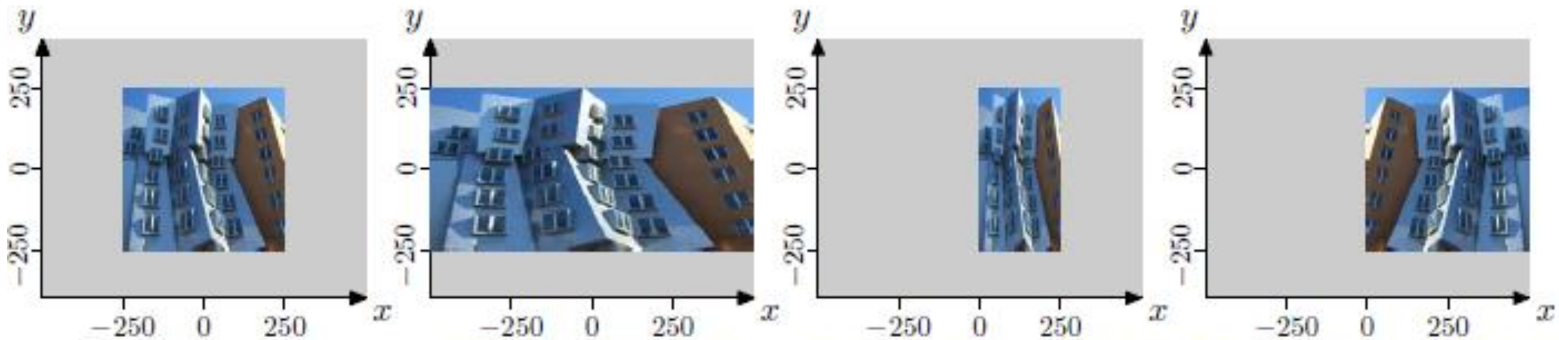


$f_1(x, y) = f(2x, y)$?

$f_2(x, y) = f(2x - 250, y)$?

$f_3(x, y) = f(-x - 250, y)$?

Check Yourself



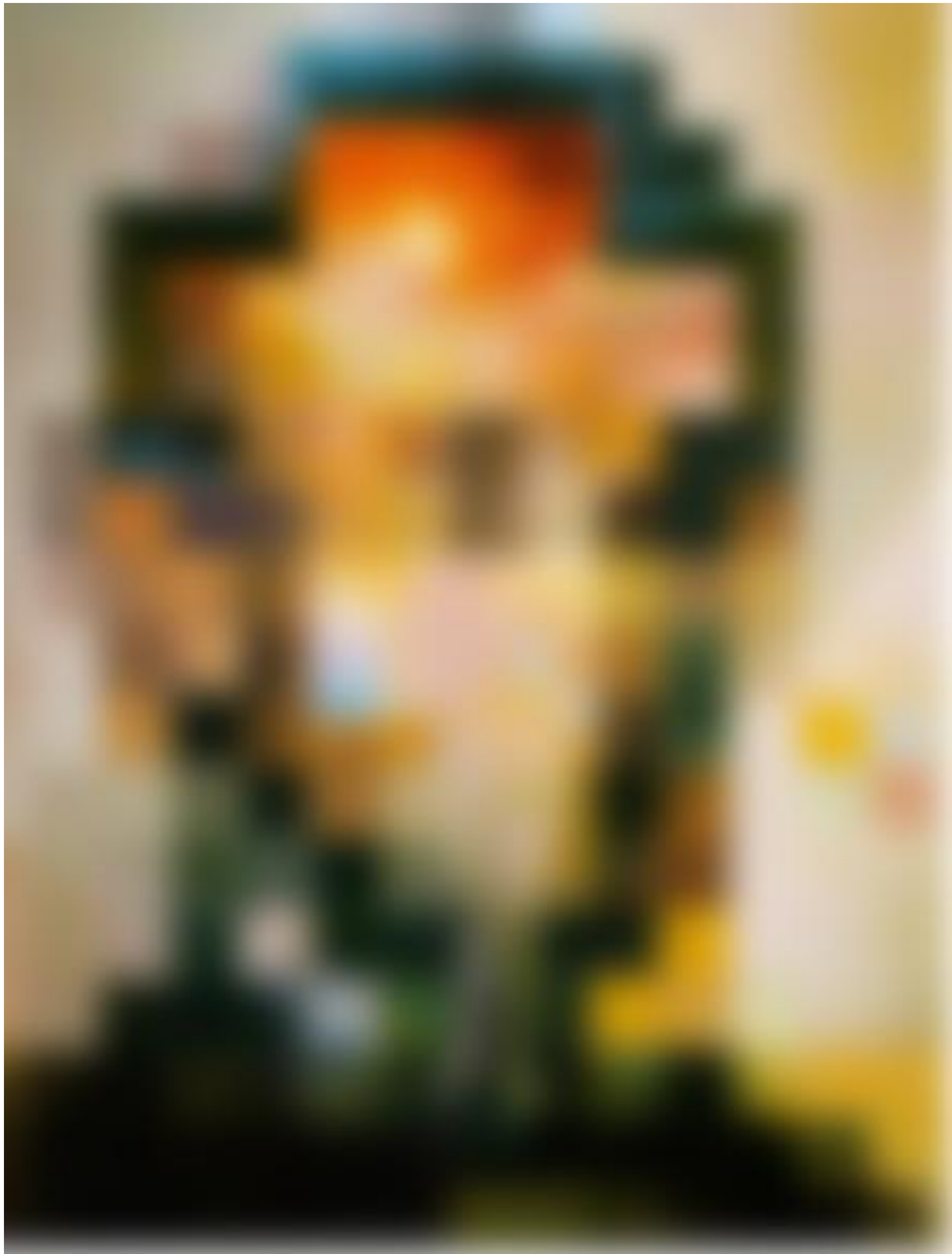
$f(x,y)$ $\quad$ $f_1(x,y) = f(2x,y)$? $\quad$ $f_2(x,y) = f(2x-250,y)$? $\quad$ $f_3(x,y) = f(-x-250,y)$?

$x = 0$	$\rightarrow f_1(0,y) = f(0,y)$	✓
$x = 250$	$\rightarrow f_1(250,y) = f(500,y)$	✗
$x = 0$	$\rightarrow f_2(0,y) = f(-250,y)$	✓
$x = 250$	$\rightarrow f_2(250,y) = f(250,y)$	✓
$x = 0$	$\rightarrow f_3(0,y) = f(-250,y)$	✗
$x = 250$	$\rightarrow f_3(250,y) = f(-500,y)$	✗

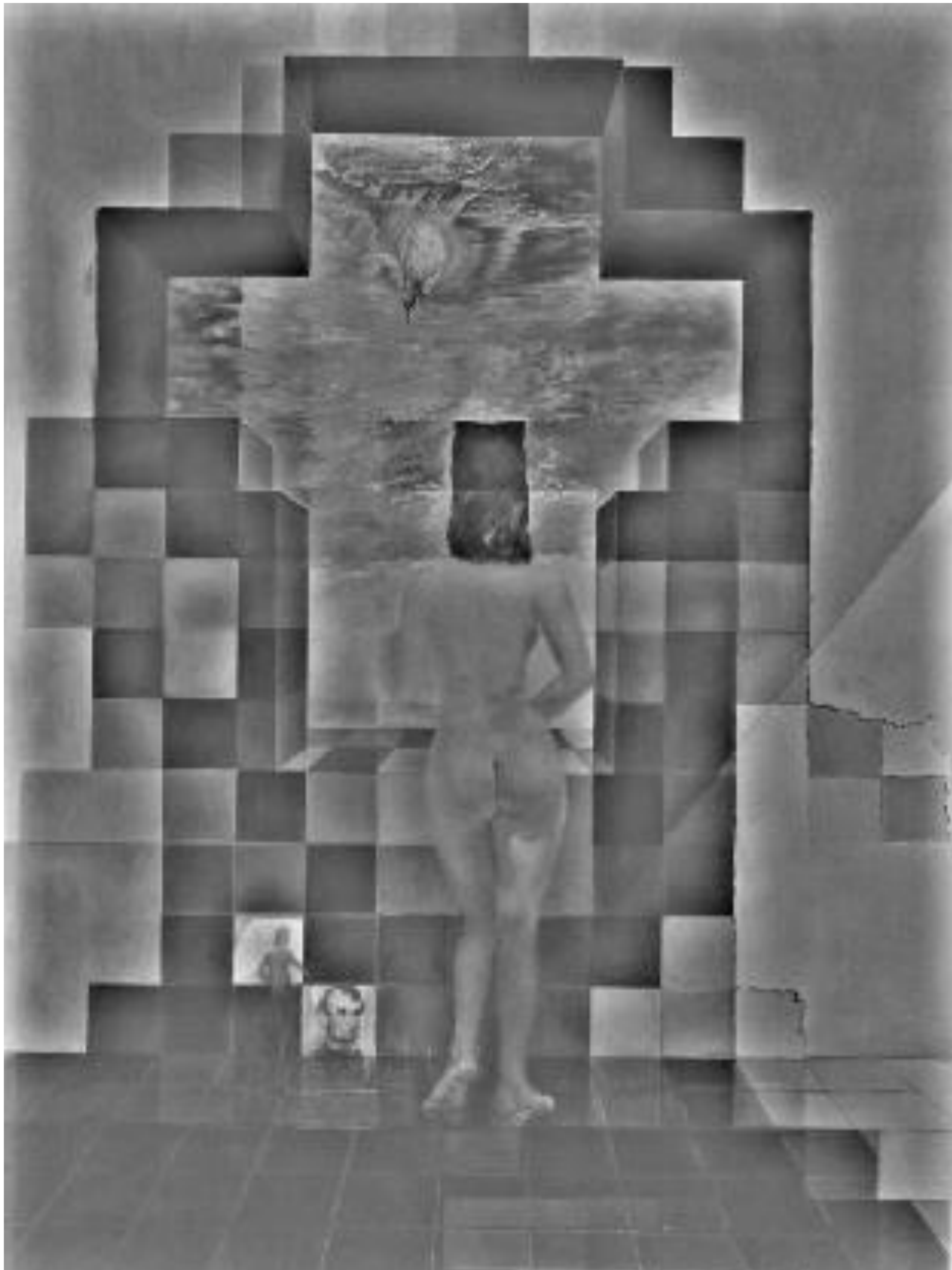
Frequency

Salvador Dali
"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln", 1976

Frequency Cues

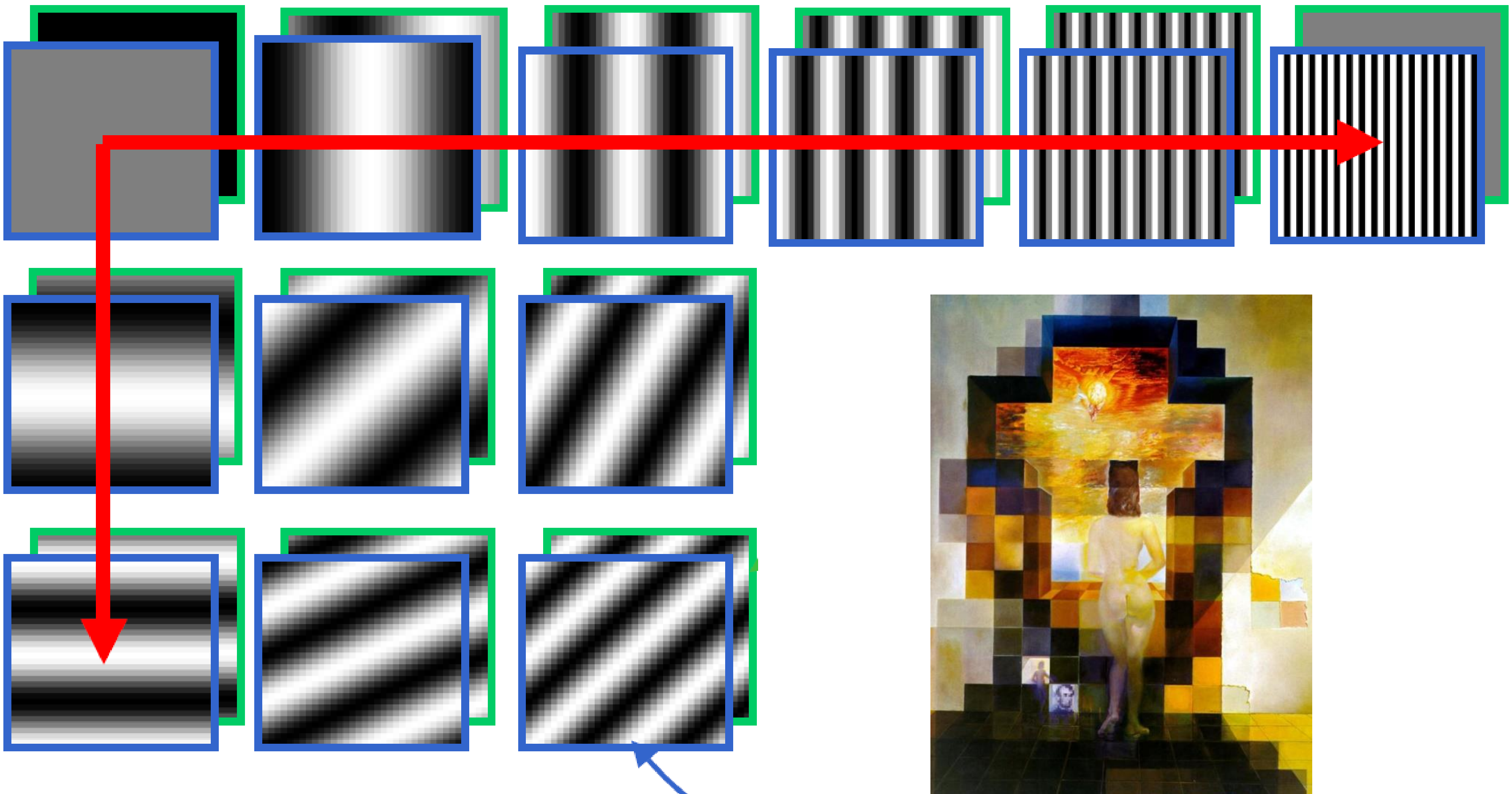


Frequency Cues



A nice set of basis

Teases away fast vs. slow changes in the image.



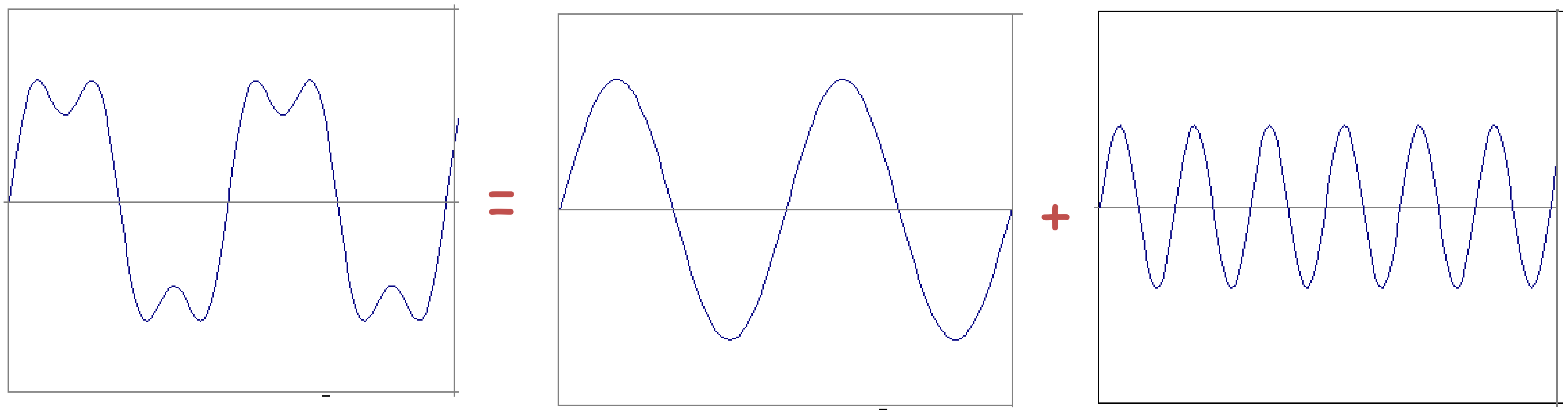
This change of basis has a special name…

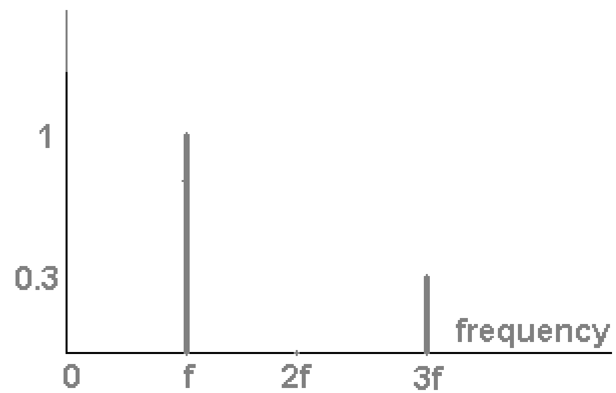
Jean Baptiste Joseph Fourier (1768-1830)

- had crazy idea (1807):
- ***Any*** *periodic function can be rewritten as a weighted sum of sines and cosines of different frequencies.*
- Don't believe it?
  - Neither did Lagrange, Laplace, Poisson and other big wigs
  - Not translated into English until 1878!
- But it's true!
  - called Fourier Series

Frequency Spectra

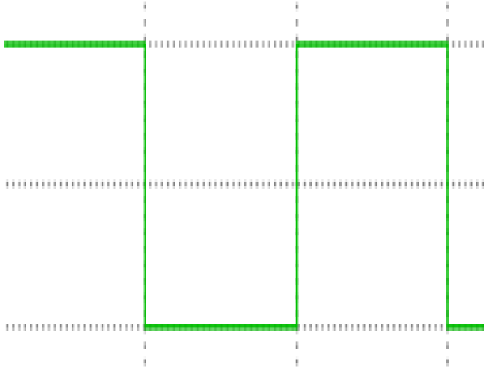
- example : $g(t) = \sin(2\pi f\, t) + (1/3)\sin(2\pi(3f)\, t)$



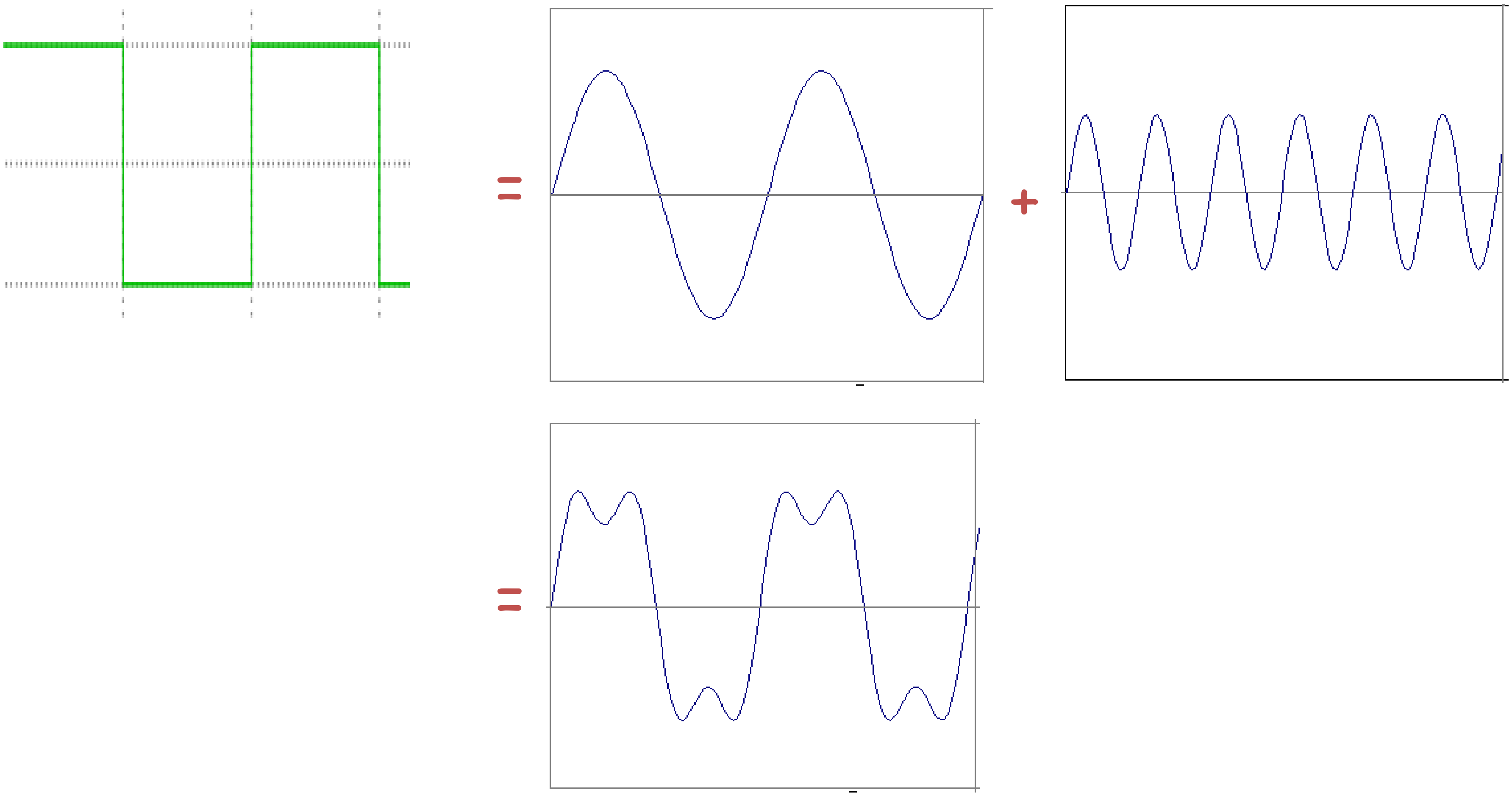


Slides: Efros

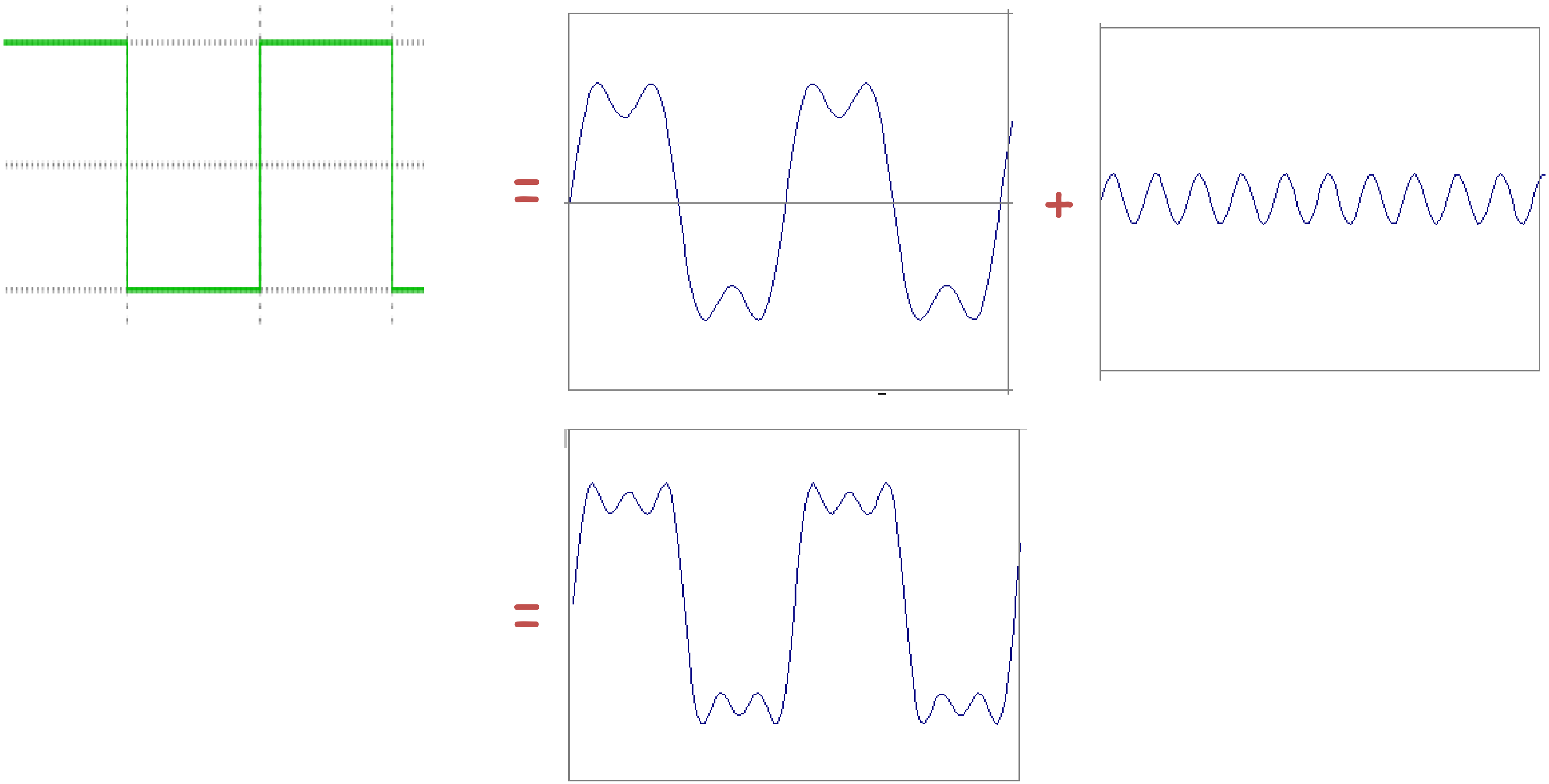
Frequency Spectra



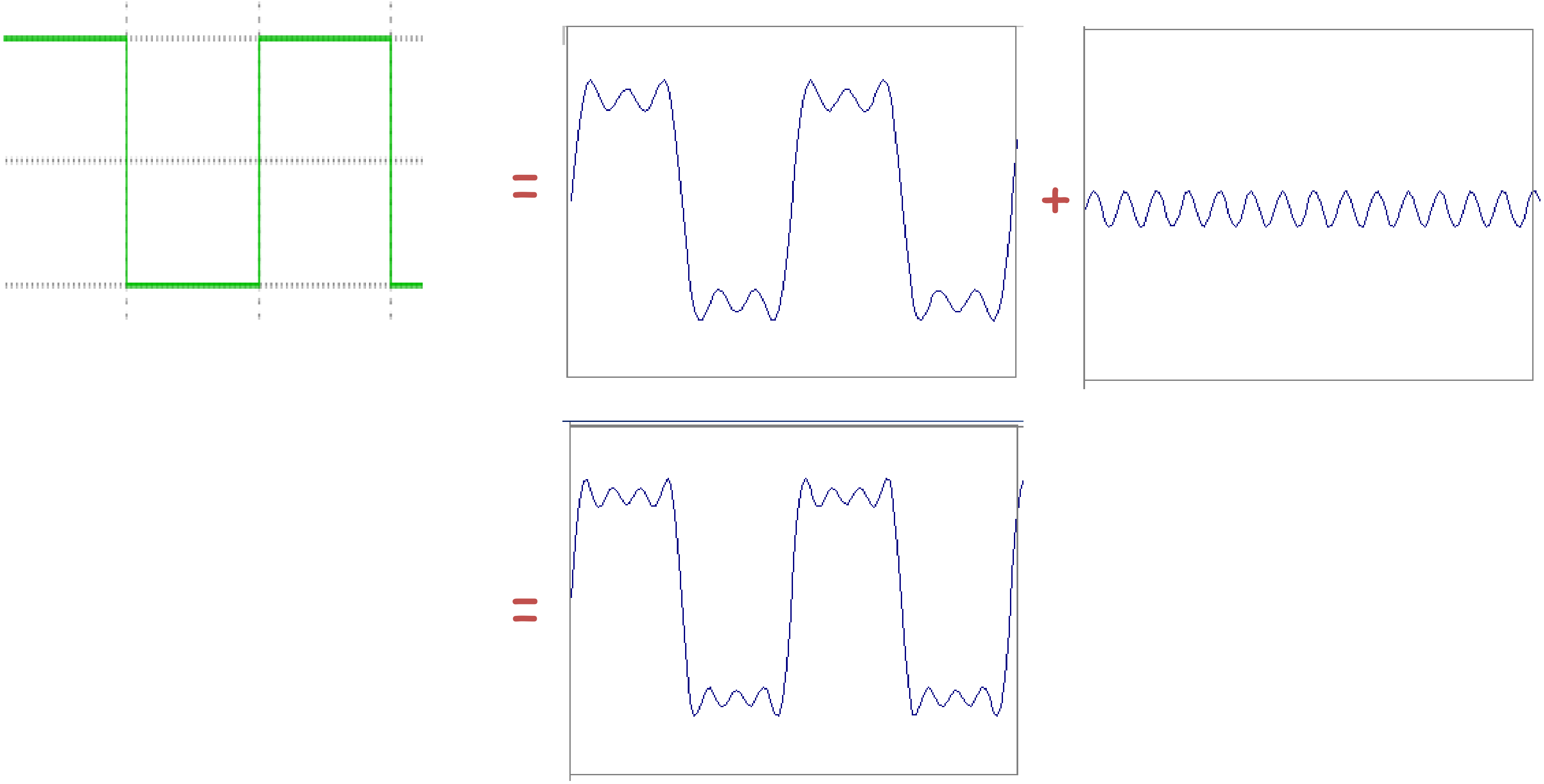
Frequency Spectra



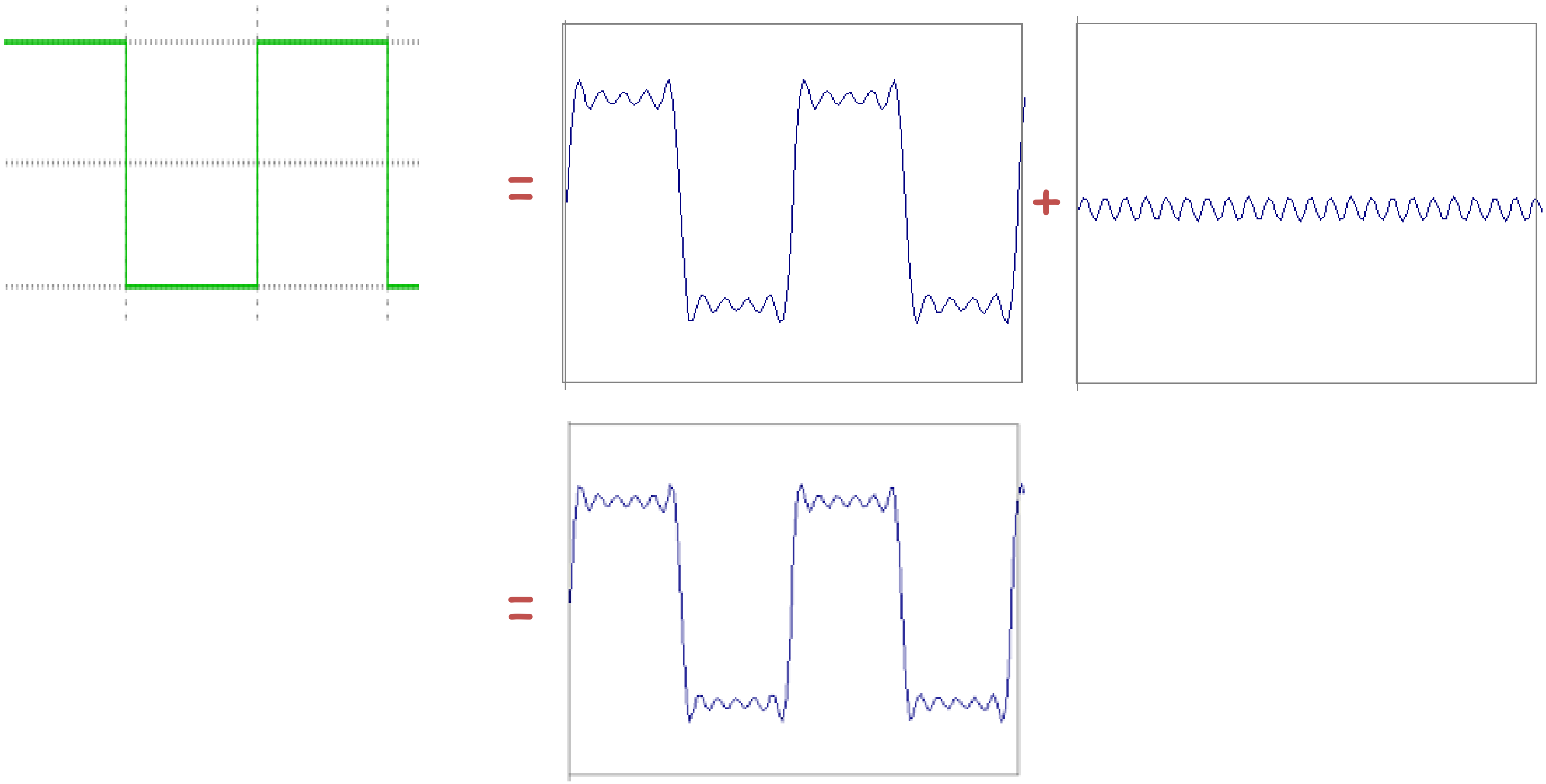
Frequency Spectra



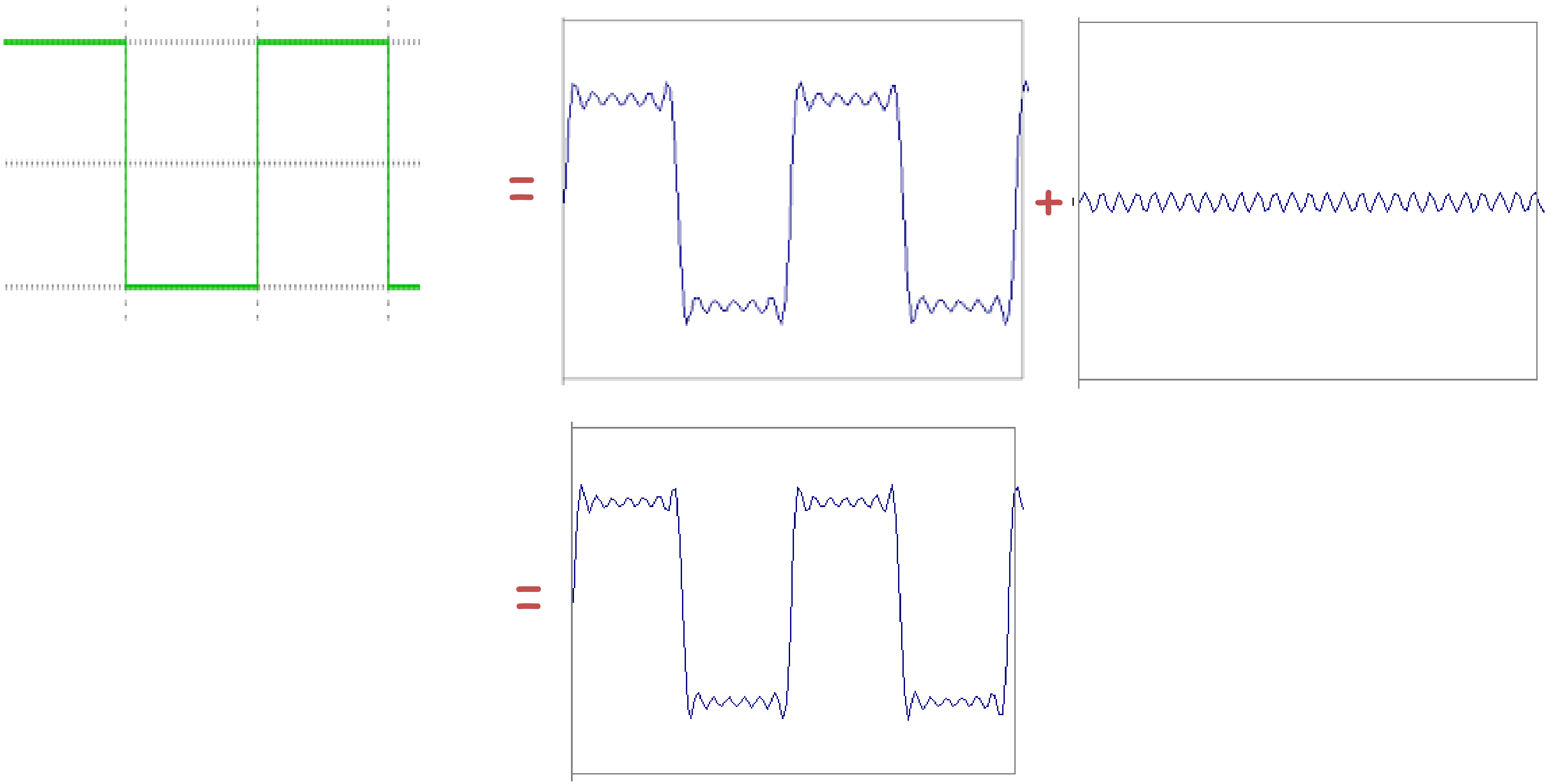
Frequency Spectra



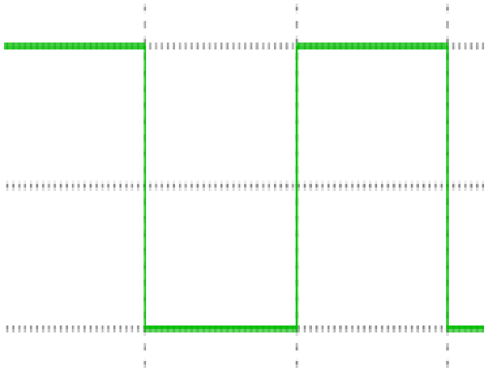
Frequency Spectra



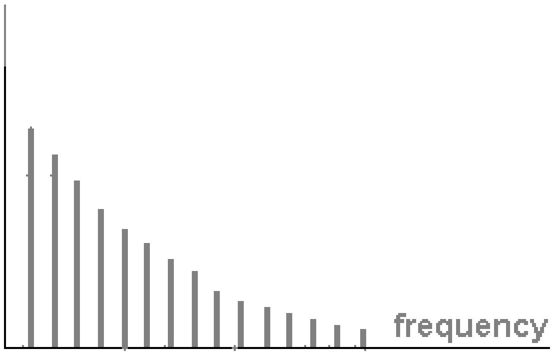
Frequency Spectra



Frequency Spectra

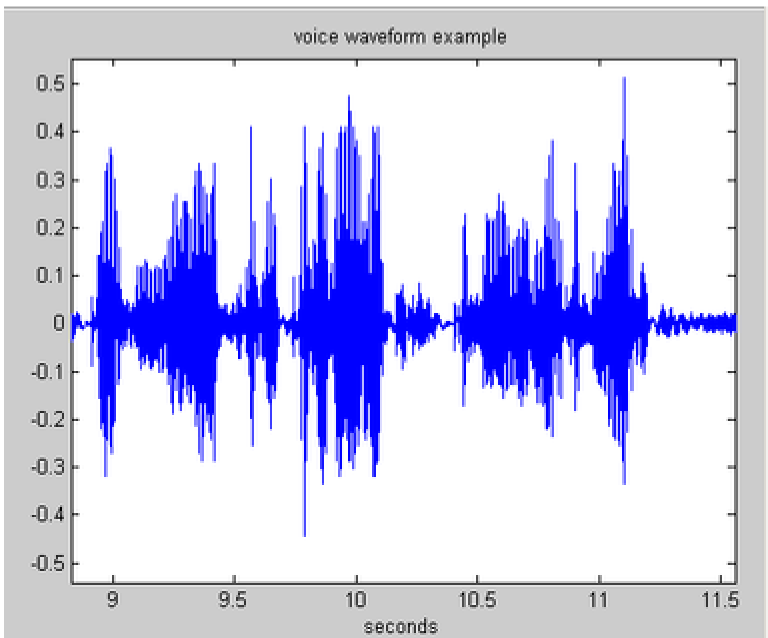


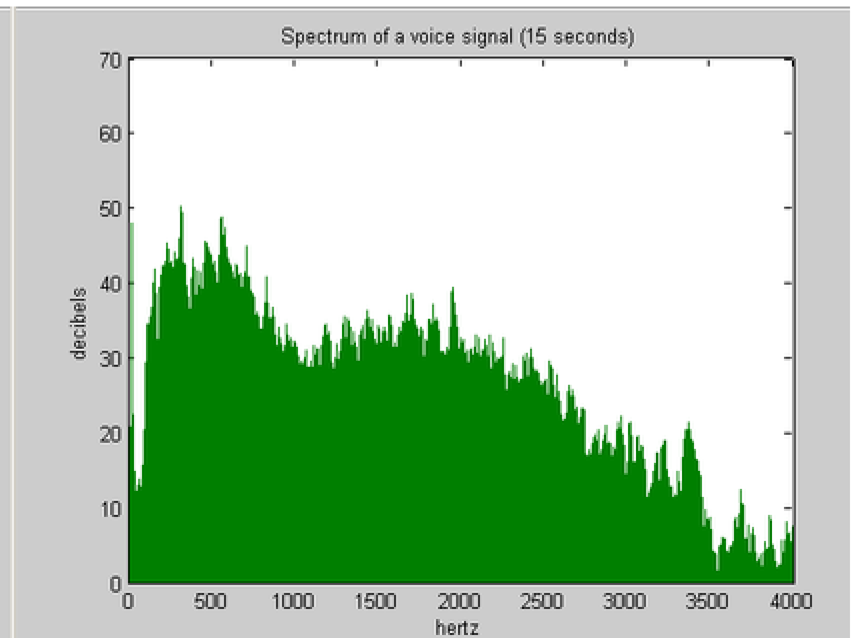
$$= \quad A\sum_{k=1}^{\infty}\frac{1}{k}\sin(2\pi kt)$$



Example: Music

- We think of music in terms of frequencies at different magnitudes



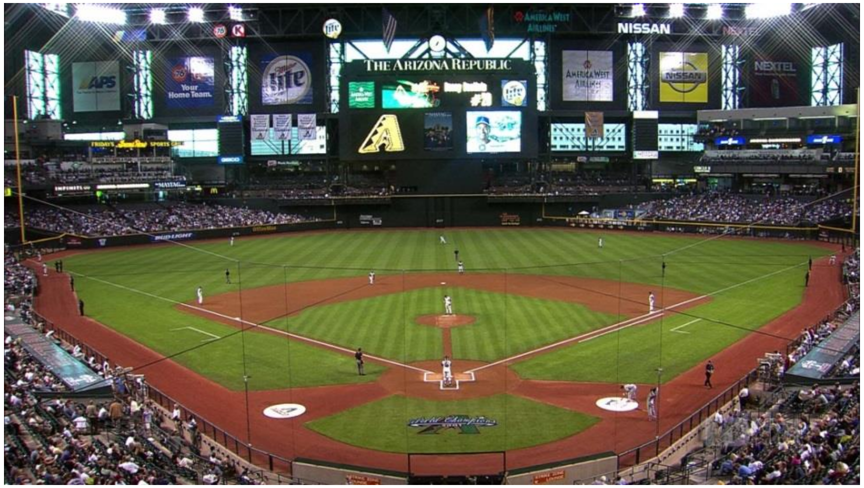


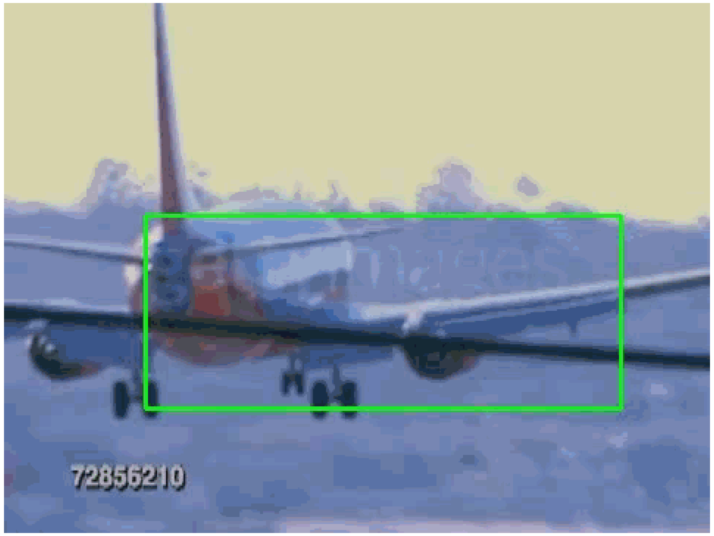
Slide: Hoiem

Human Hearing and Voice Signals

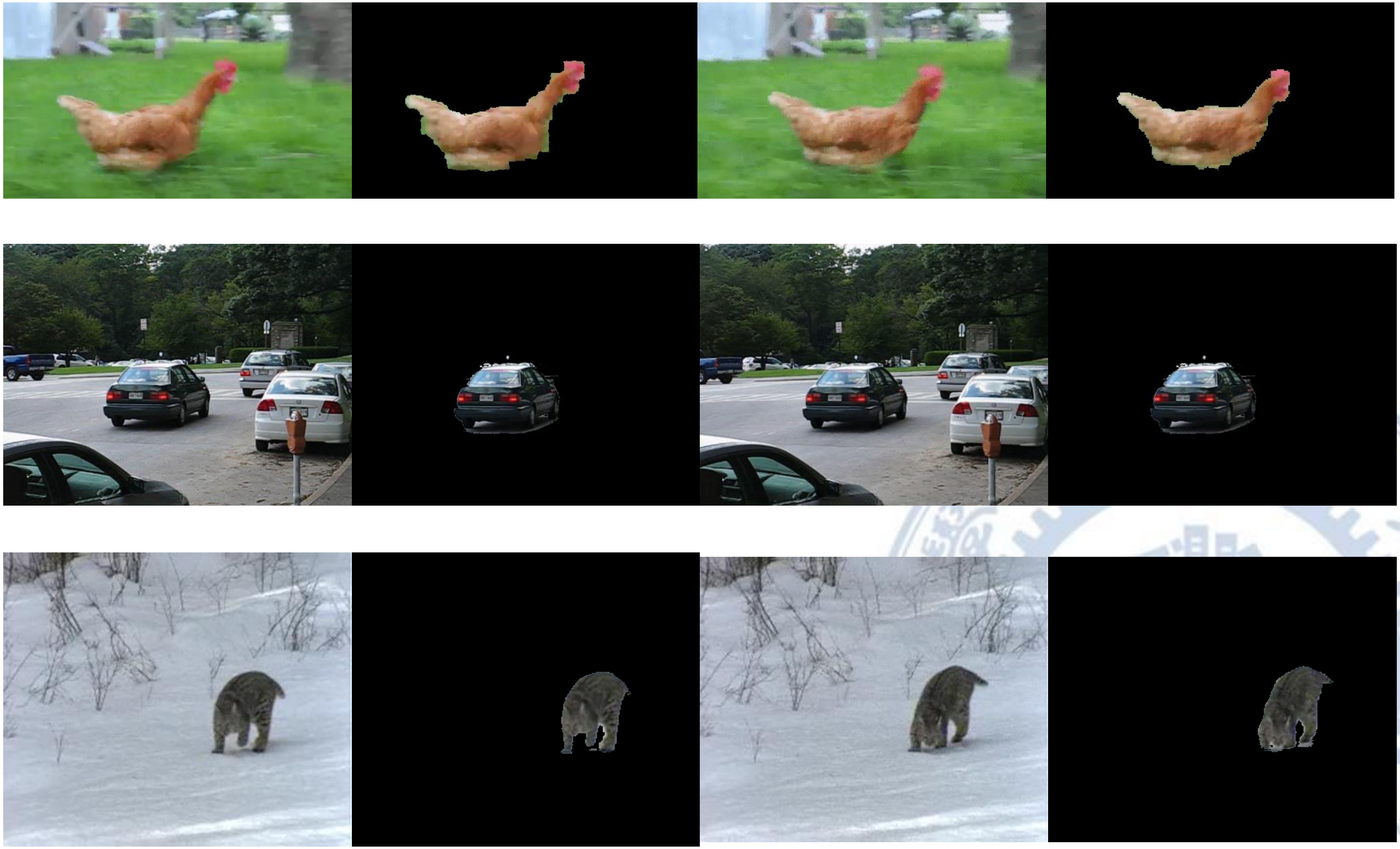
- Range is about 20 Hz to 20 kHz, most sensitive at 2 - 4 KHz.
- Dynamic range (quietest to loudest) is about 96 dB
- Normal voice range is about 500 Hz to 2 kHz
  - Low frequencies are vowels and bass
  - High frequencies are consonants

Image, Video, Stereo Signal





Computer Vision



A variety of Image Signals

- Energy of one photon

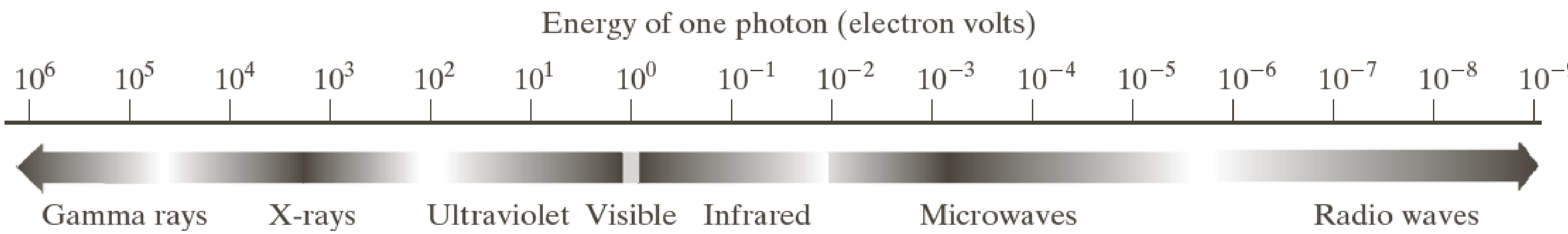
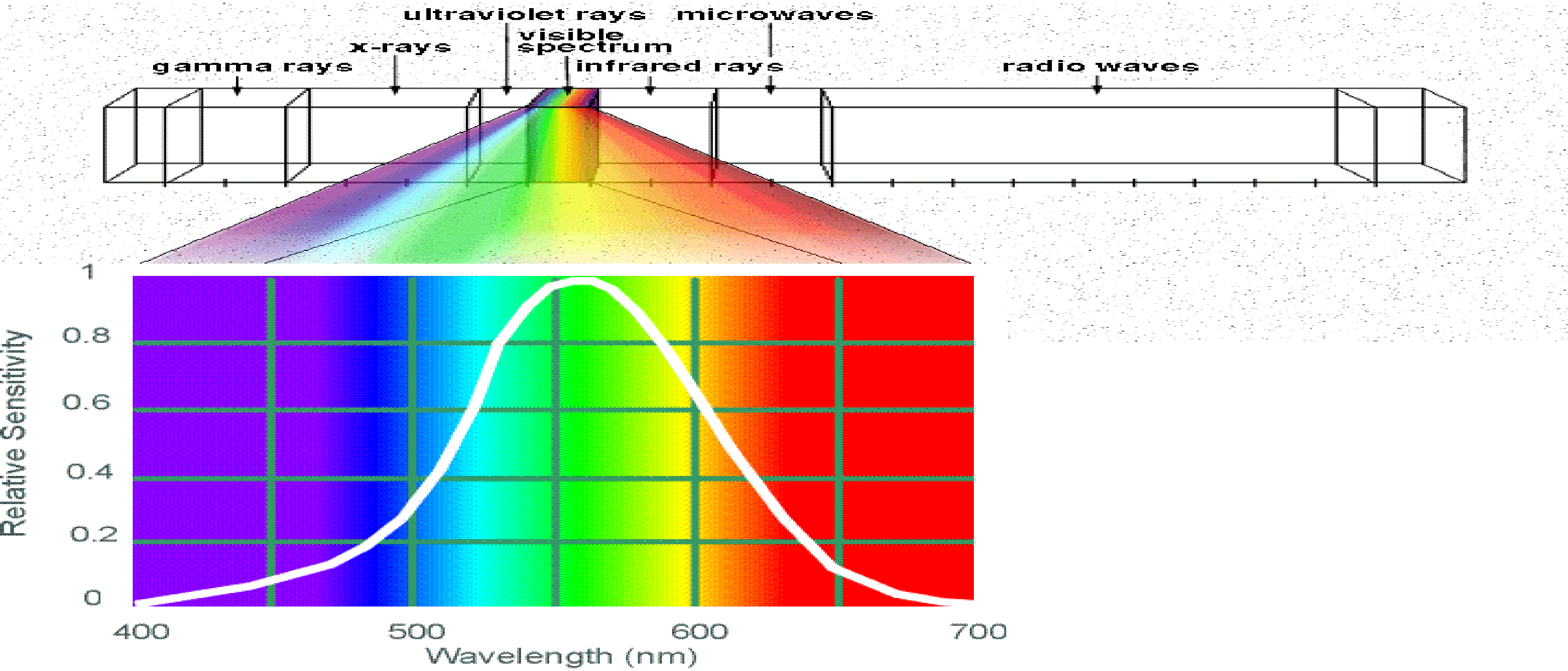


FIGURE 1.5 The electromagnetic spectrum arranged according to energy per photon.

- Image from Invisible light
  - γ- ray imaging
  - X- ray imaging
  - Imaging in the ultraviolet band
  - Imaging in the infrared band
  - Imaging in the microwave band
  - Imaging in the radio band

Electromagnetic Spectrum

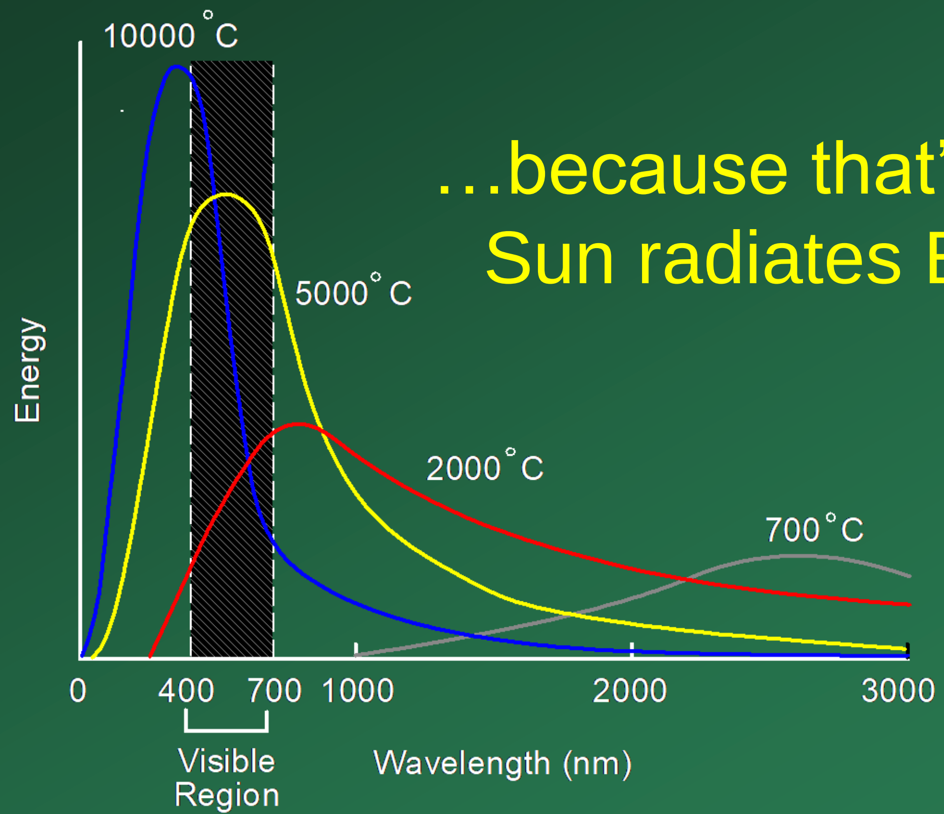


Human Luminance Sensitivity Function

http://www.yorku.ca/eye/photopik.htm

Visible Light

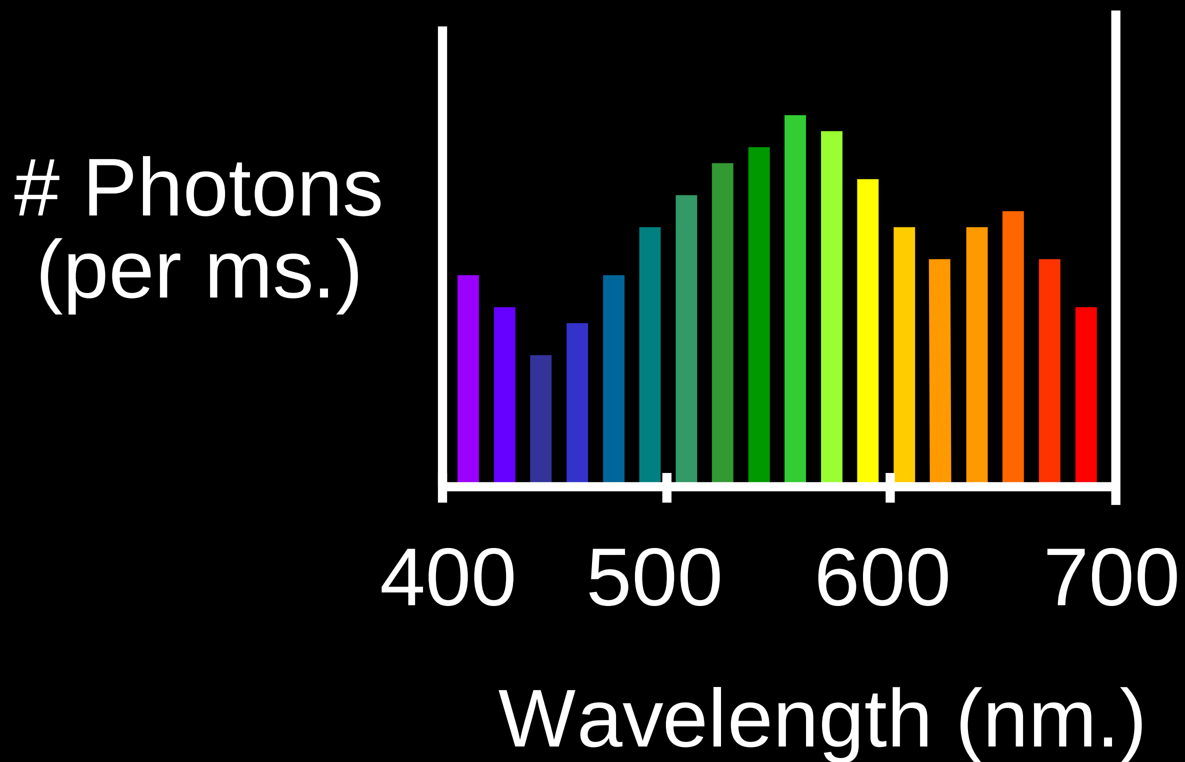
Why do we see light of these wavelengths?



…because that's where the Sun radiates EM energy

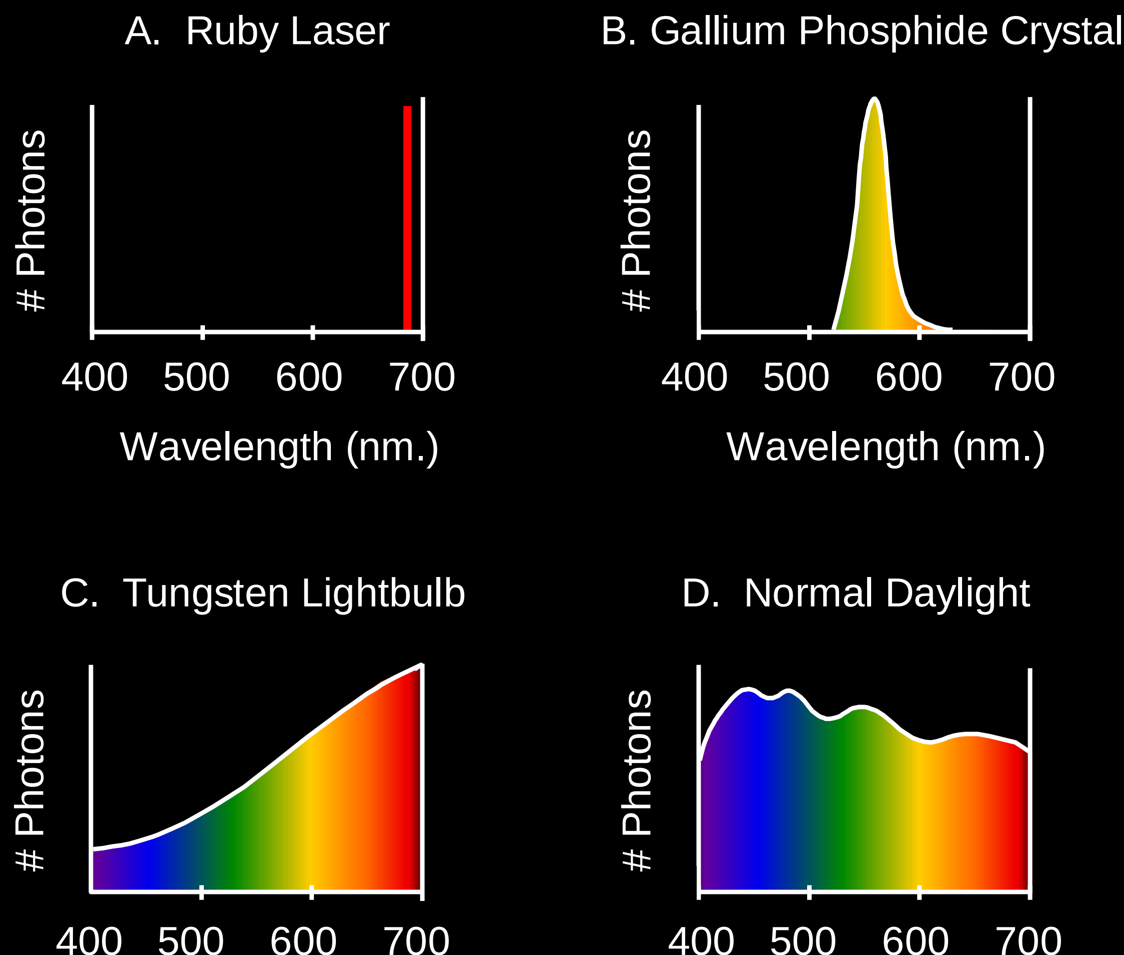
The Physics of Light

Any patch of light can be completely described physically by its spectrum: the number of photons (per time unit) at each wavelength 400 - 700 nm.



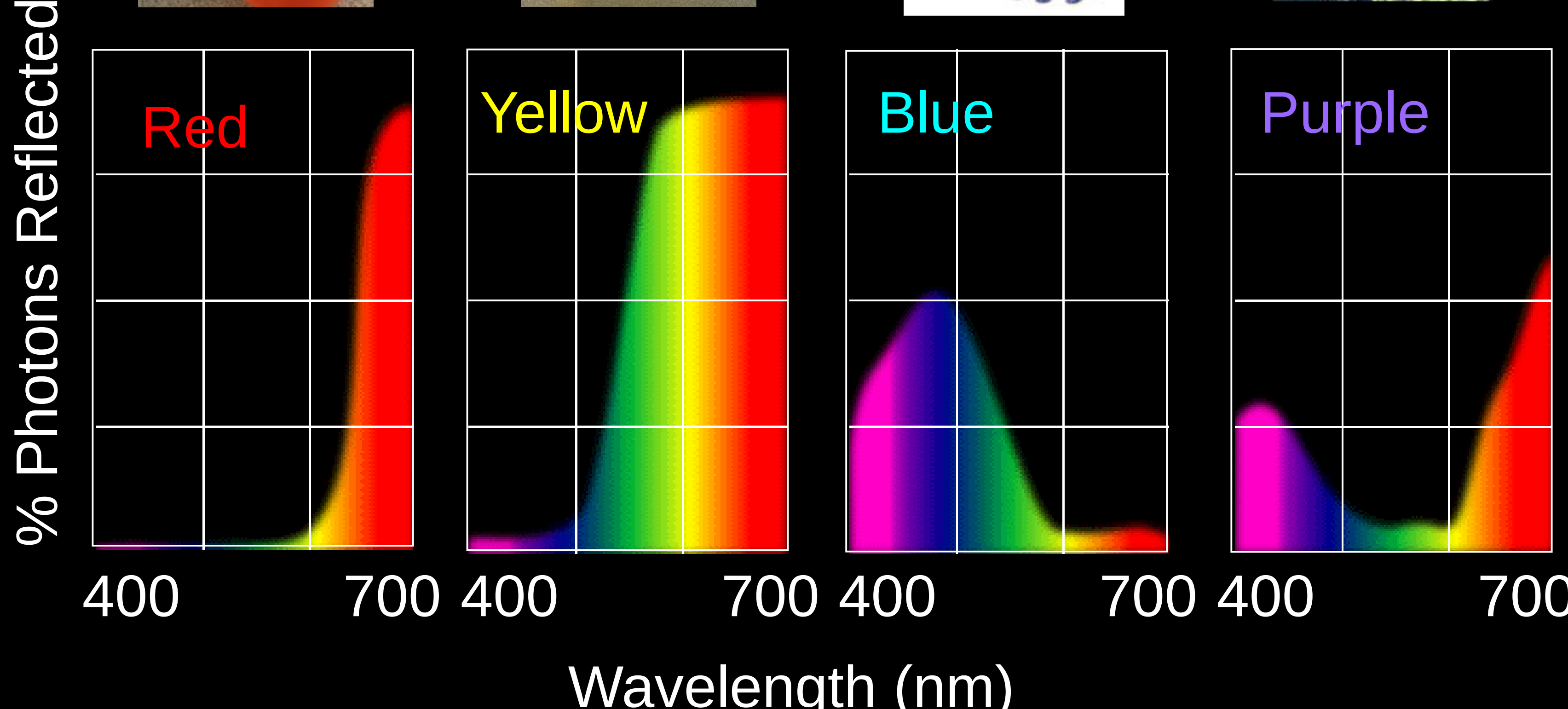
The Physics of Light

Some examples of the spectra of light sources



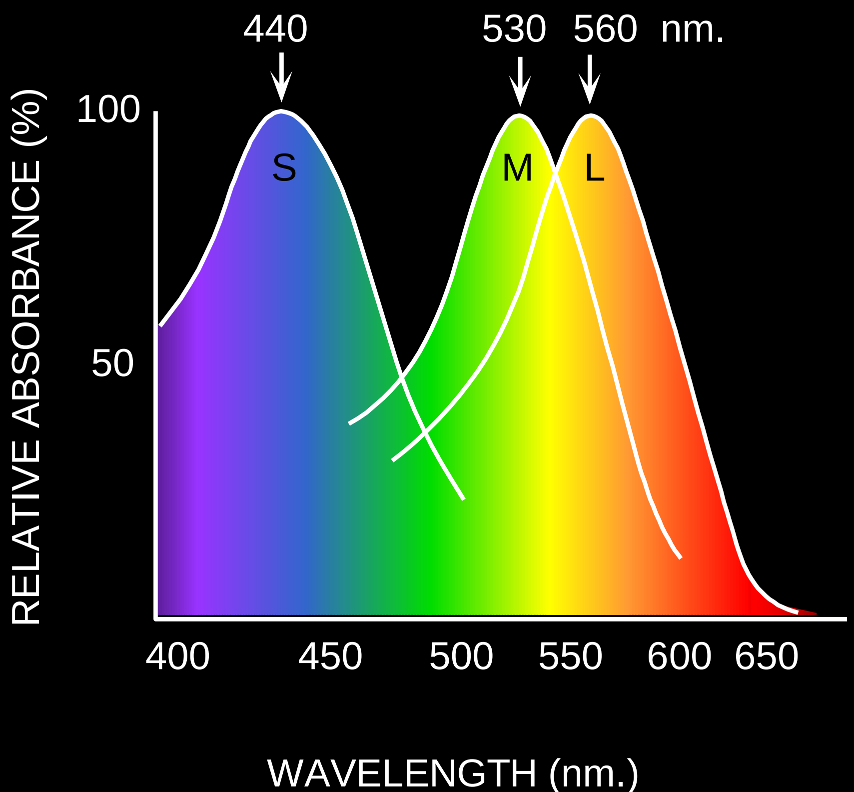
The Physics of Light

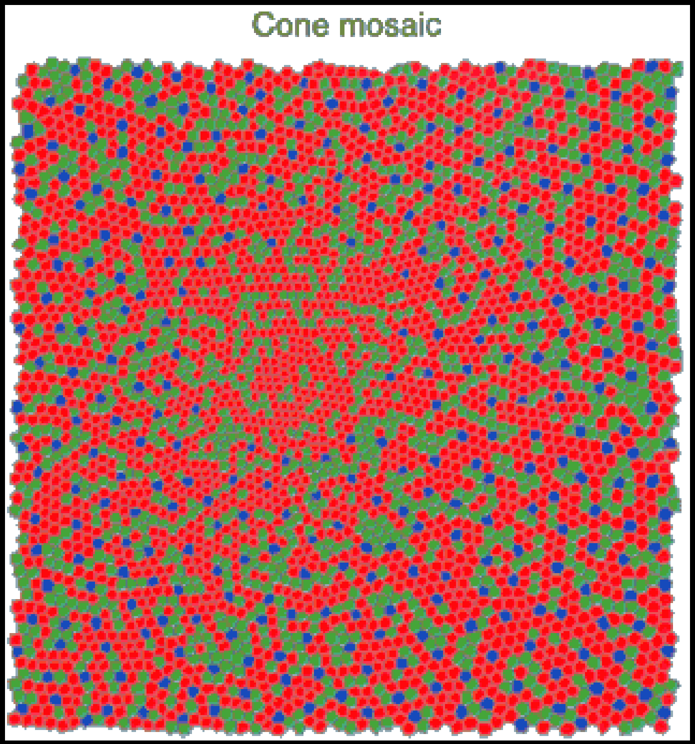
Some examples of the reflectance spectra of surfaces



Physiology of Color Vision

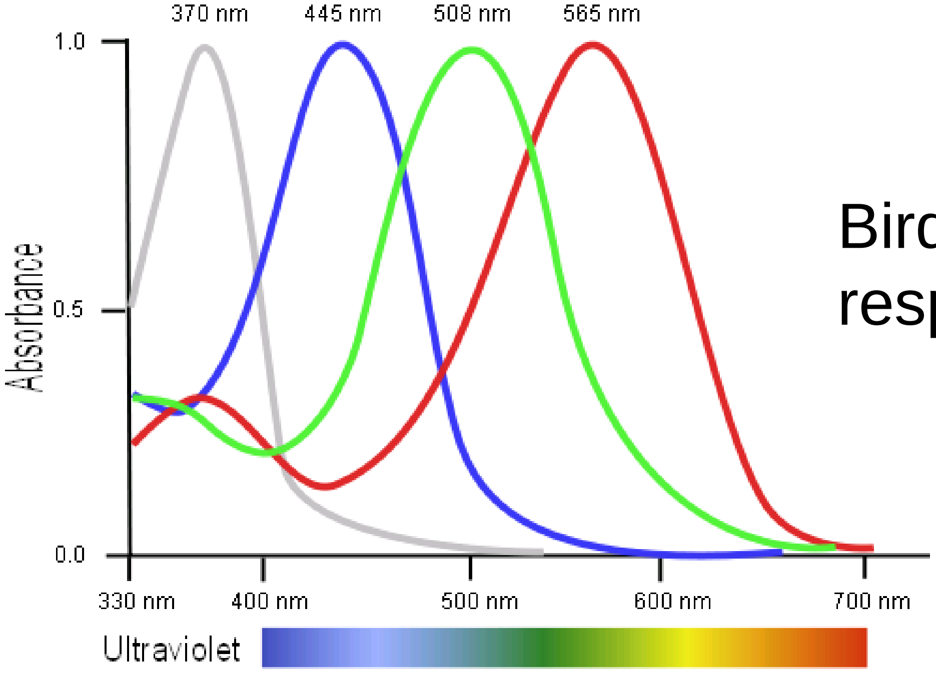
Three kinds of cones:





- Why are M and L cones so close?
- Why are there 3?

Tetrachromatism

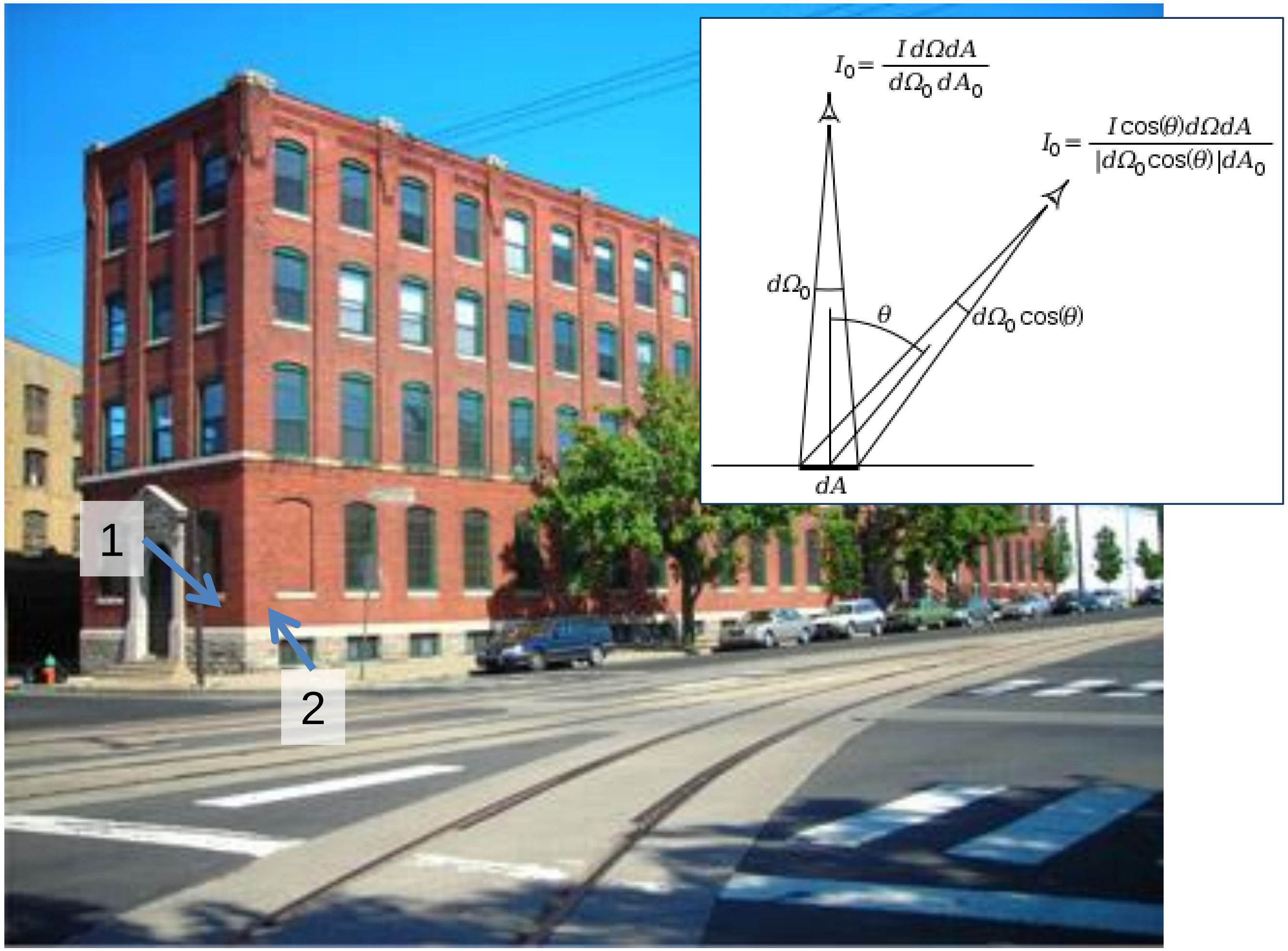


Bird cone responses

Most birds, and many other animals, have cones for ultraviolet light.

Some humans, mostly female, seem to have slight tetrachromatism.

Surface orientation and light intensity

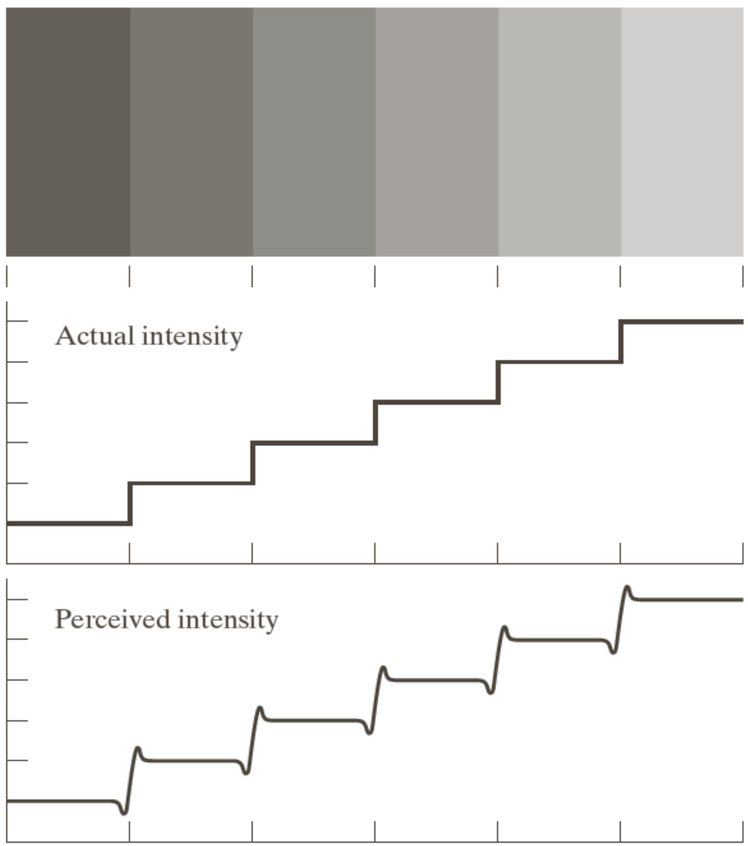


Why is (1) darker than (2)?

For diffuse reflection, will intensity change when viewing angle changes?

Visual Perception

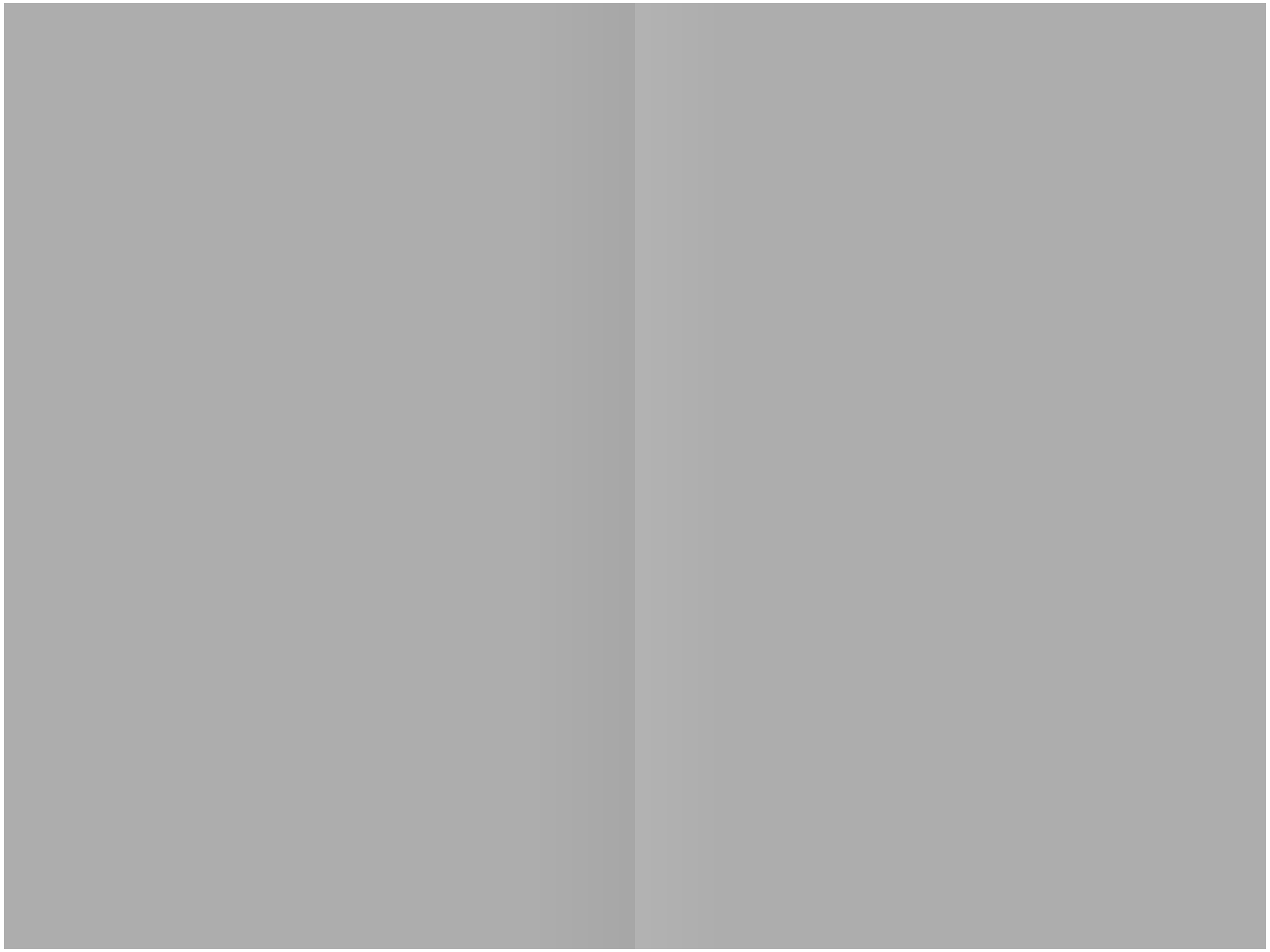
- Mach Bands



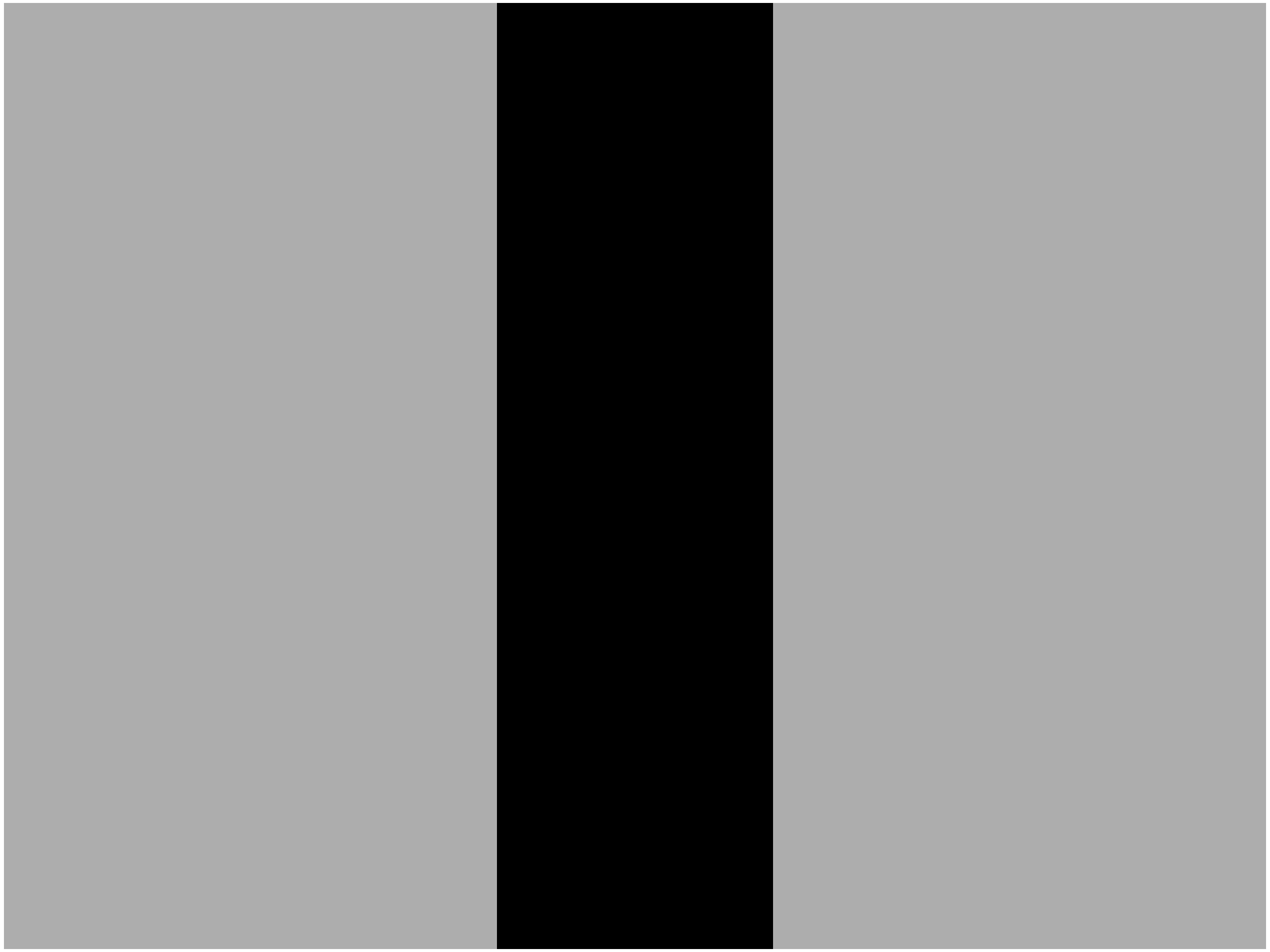
a
b
c

FIGURE 2.7 Illustration of the Mach band effect. Perceived intensity is not a simple function of actual intensity.

Visual Perception



Visual Perception



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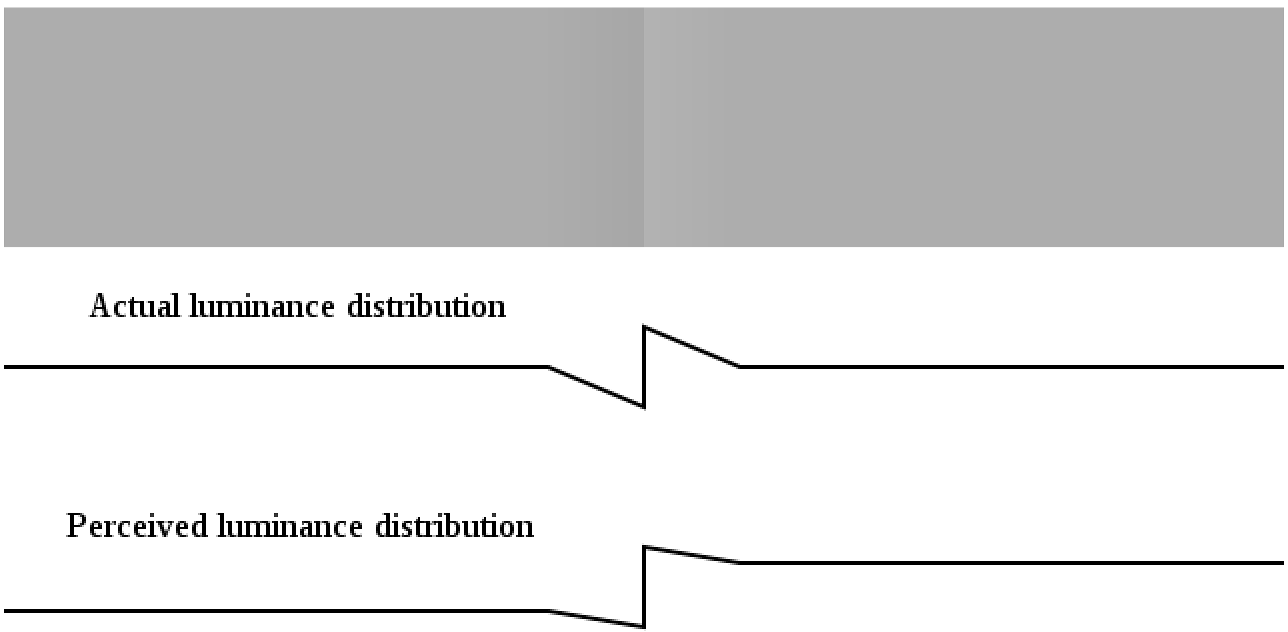
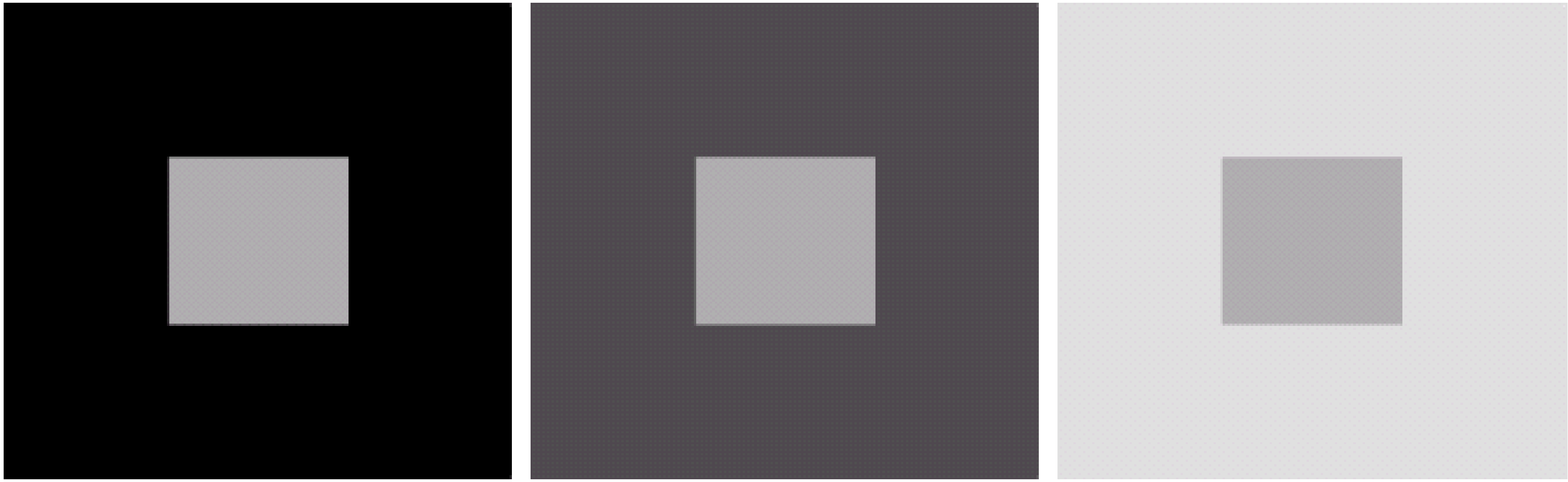
Visual Perception

- Cornsweet illusion

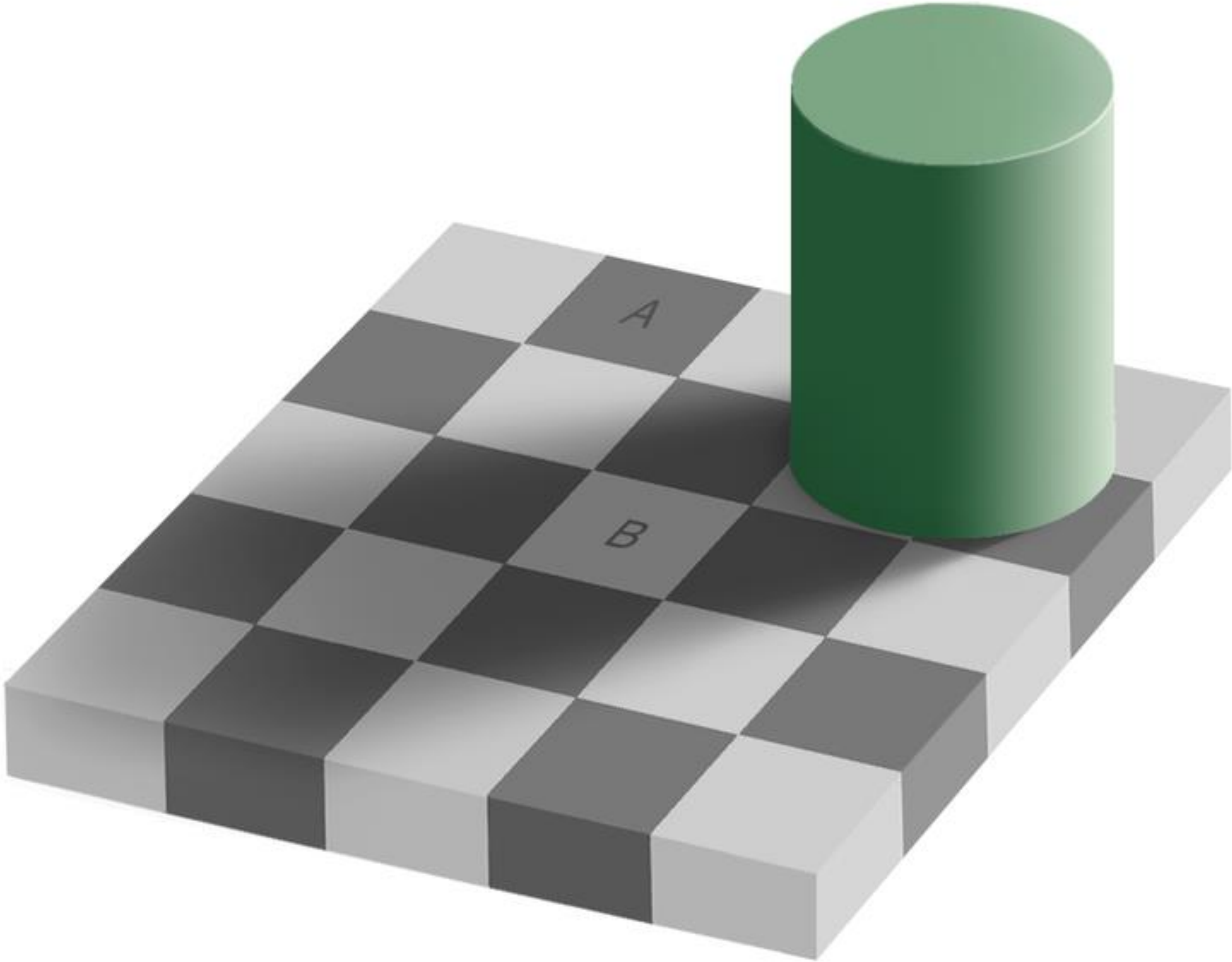


Visual Perception



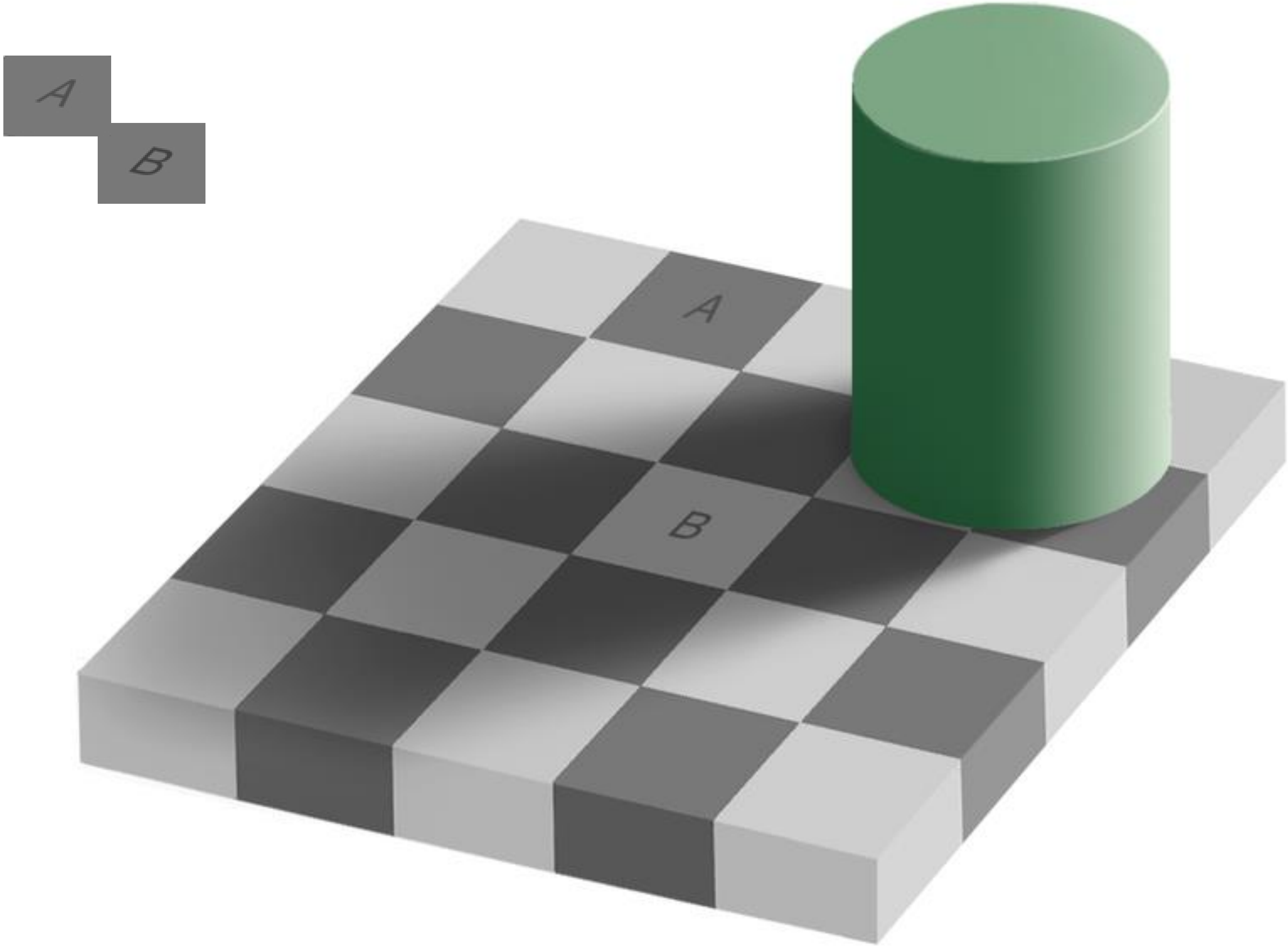
An example of *simultaneous contrast*

Perception of Intensity



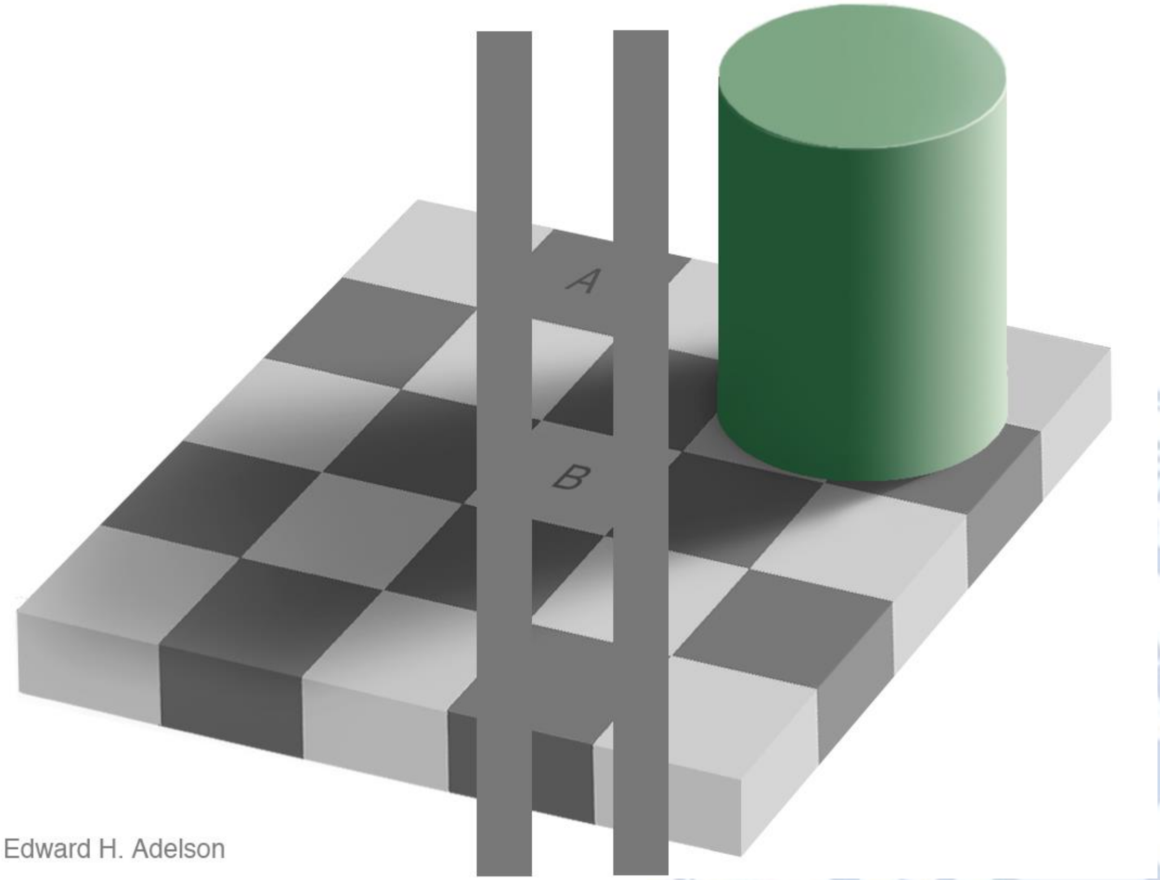
from Ted Adelson

Perception of Intensity

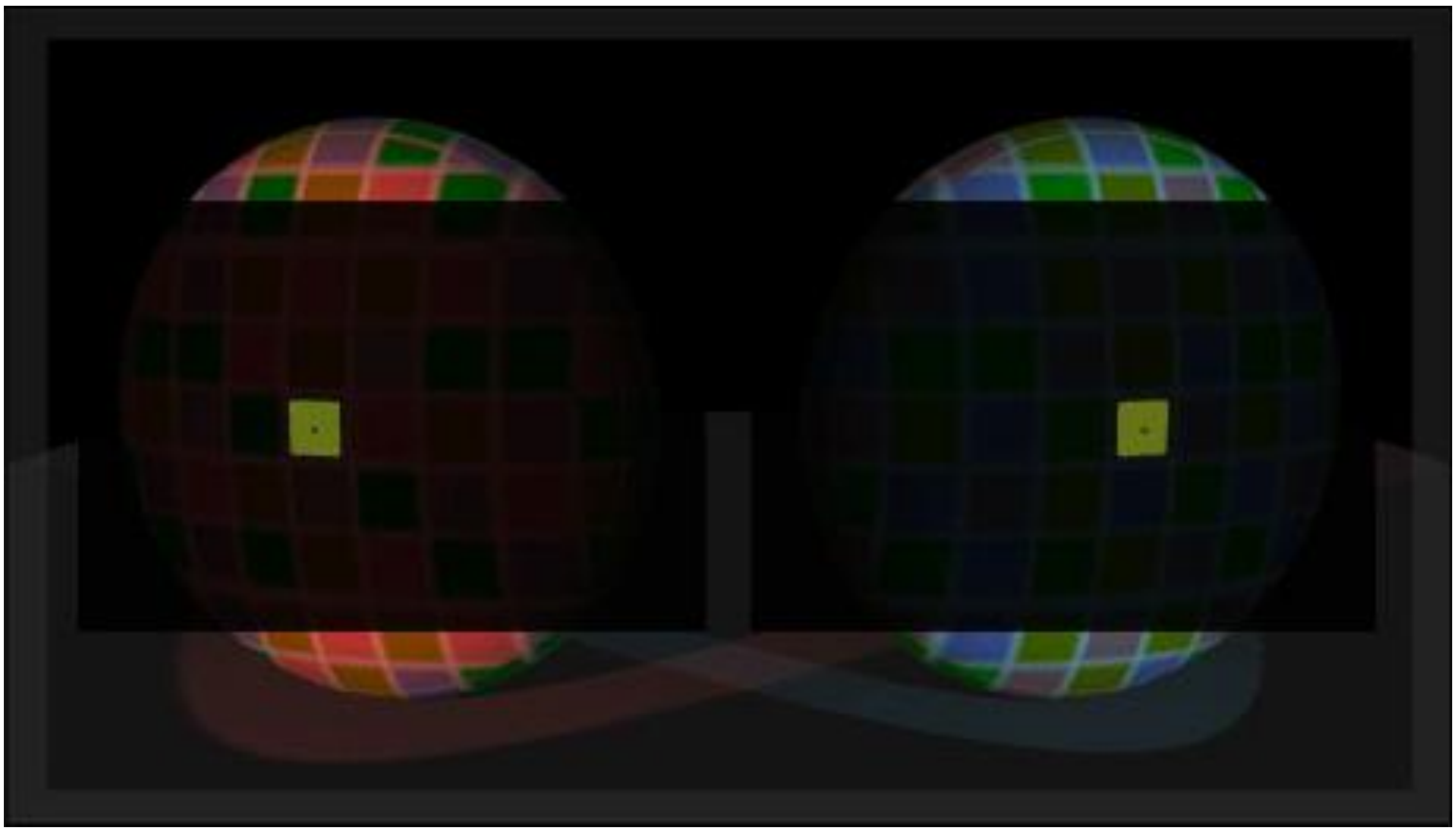


from Ted Adelson

Visual Perception

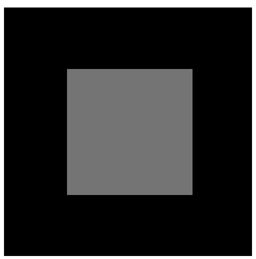


Visual Perception



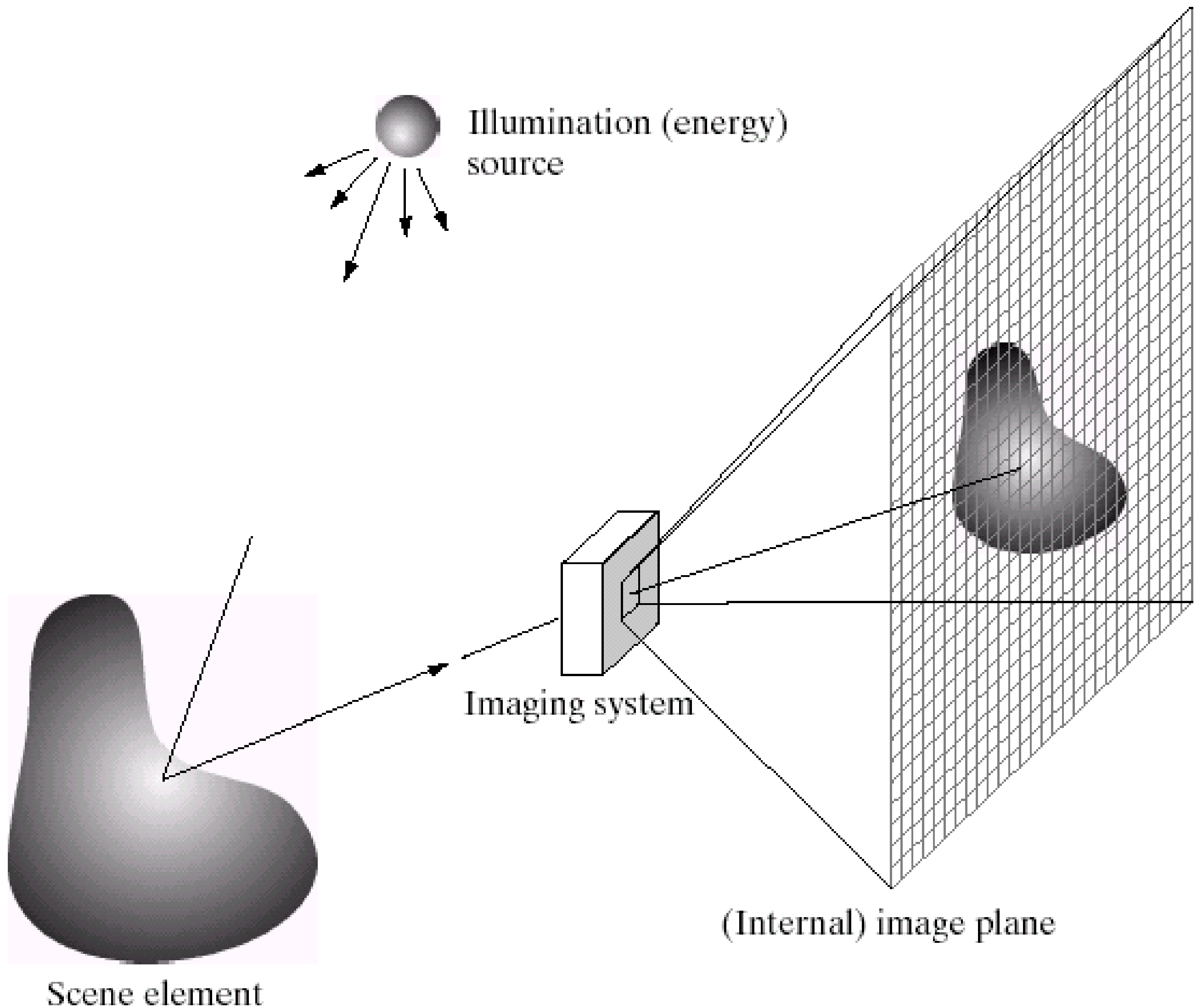
Visual Perception

- Eye is not a photometer!



- *"Every light is a shade, compared to the higher lights, till you come to the sun; and every shade is a light, compared to the deeper shades, till you come to the night."*
- — John Ruskin, 1879

Image Formation



Digital camera

A digital camera replaces film with a sensor array

- Each cell in the array is light-sensitive diode that converts photons to electrons
- Two common types: Charge Coupled Device (CCD) and CMOS
- http://electronics.howstuffworks.com/digital-camera.htm

Sensor Array

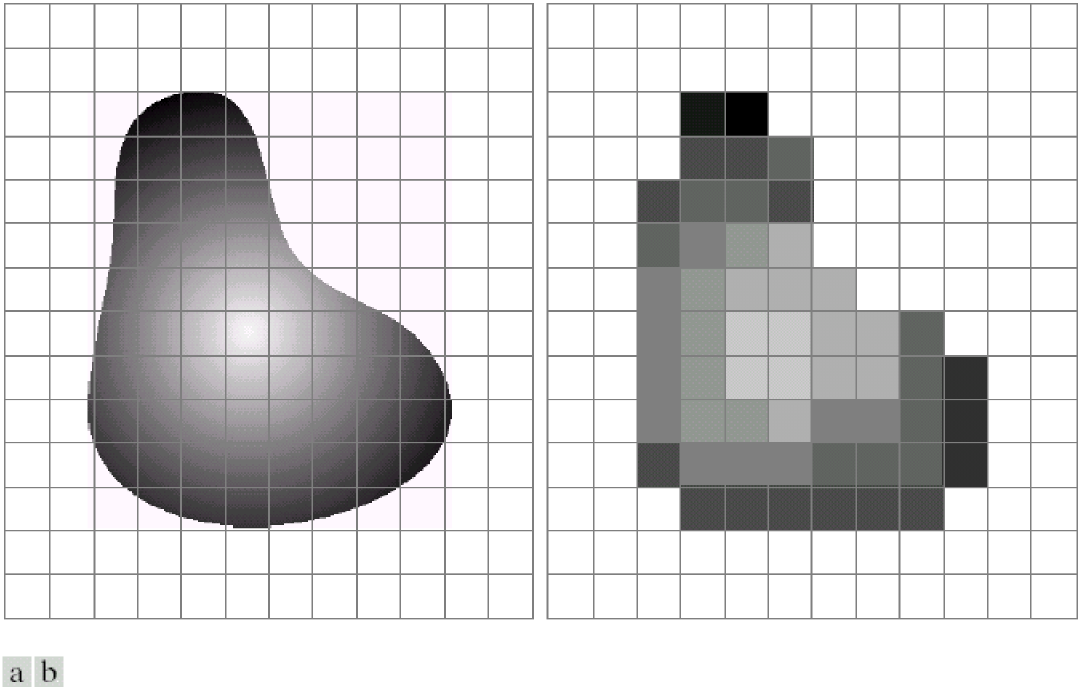
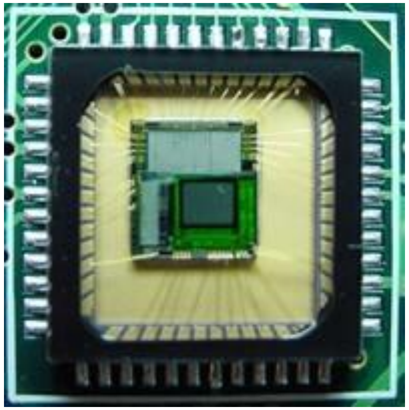
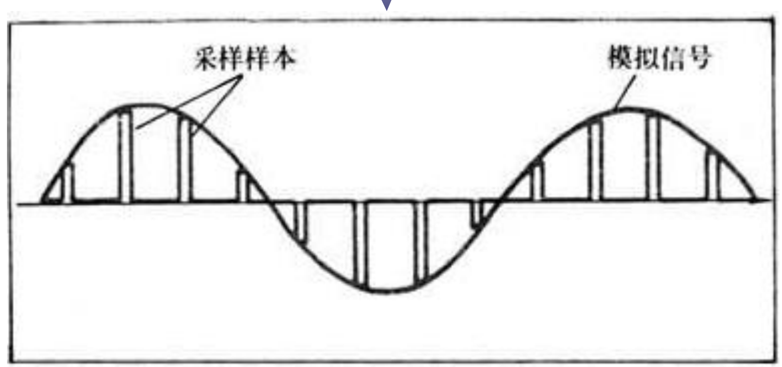


FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.



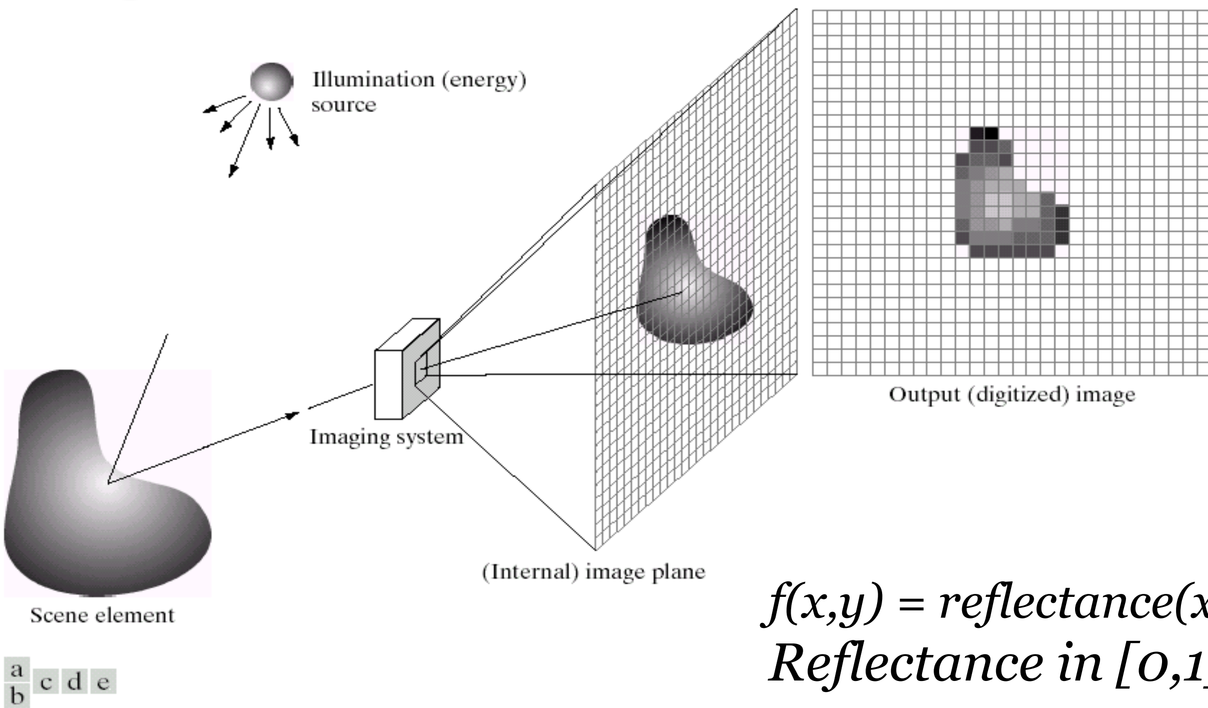
CMOS sensor

Sensing and Acquisition



Sensing and Acquisition

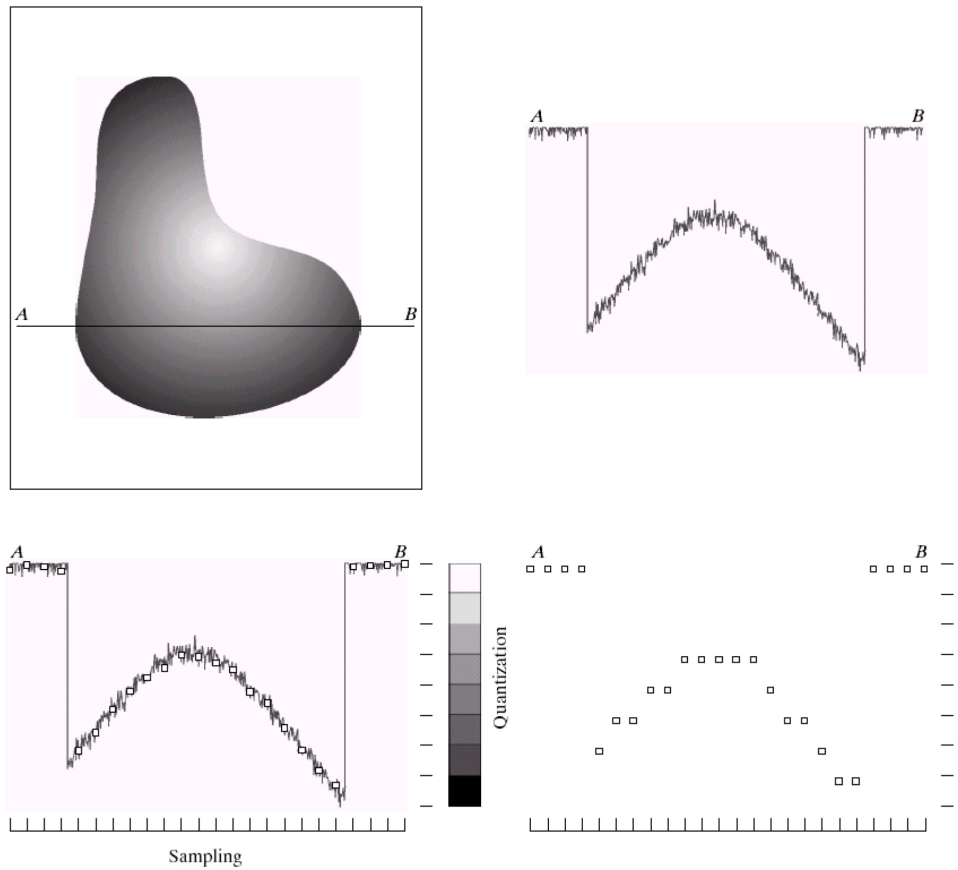
- Image Formation



*f(x,y) = reflectance(x,y) * illumination(x,y)*
Reflectance in [0,1], illumination in [0,inf]

FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Sampling and Quantization

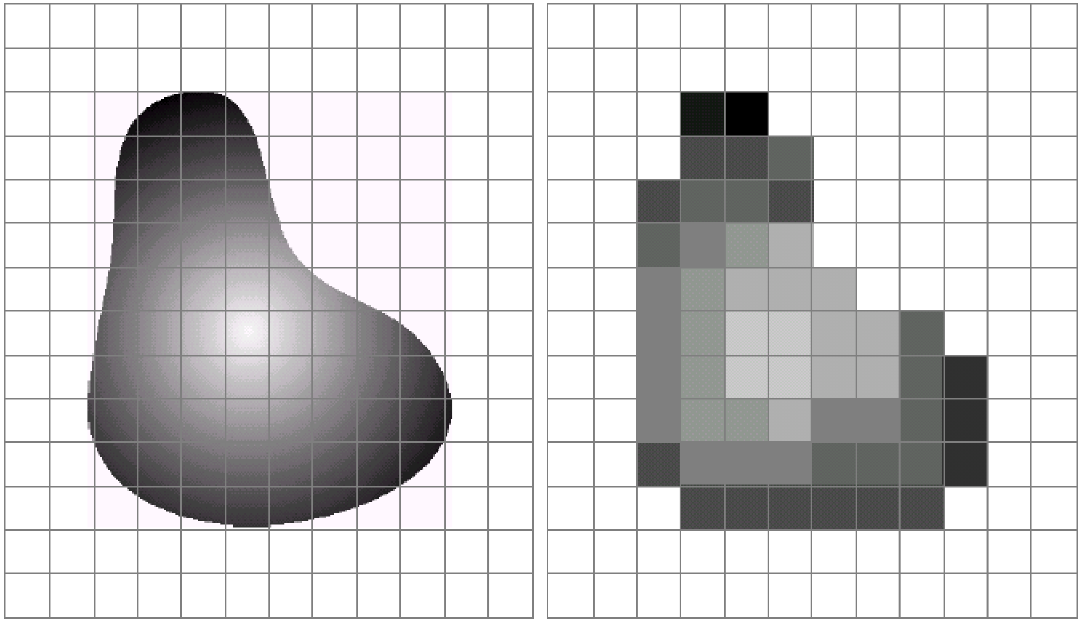


a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Sampling and Quantization

Remember that a digital image is always only an **approximation** of a real world scene



a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

Sampling and Quantization

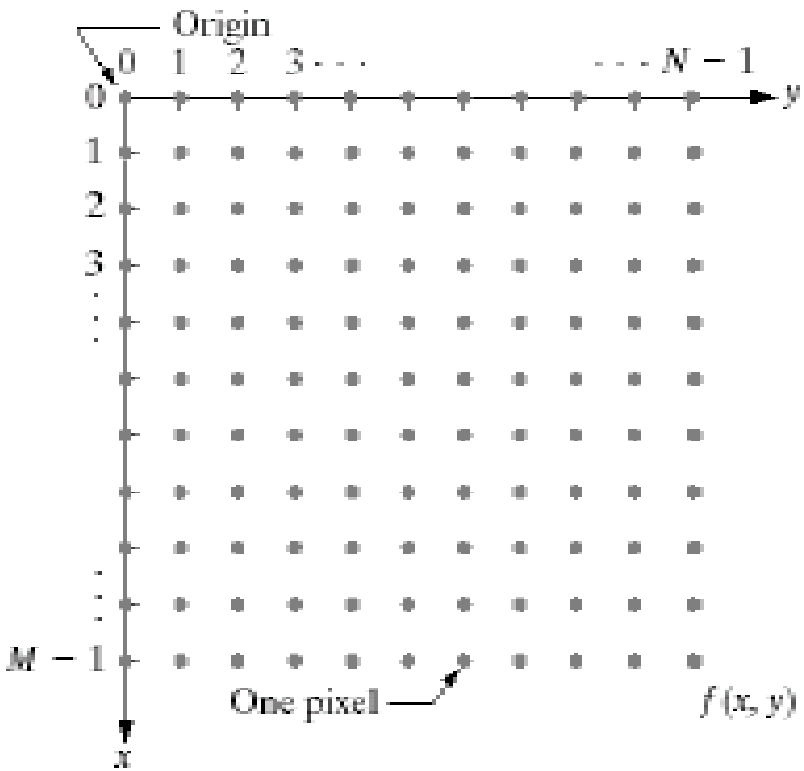
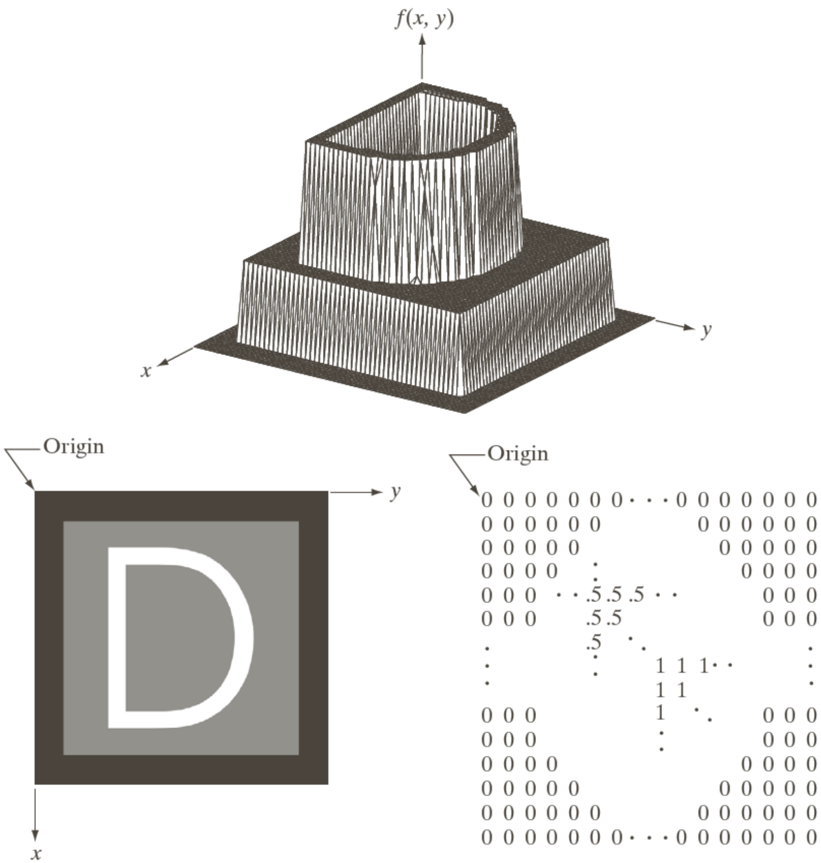


FIGURE 2.18 Coordinate convention used in this book to represent digital images.

Sampling and Quantization



a
b c

FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

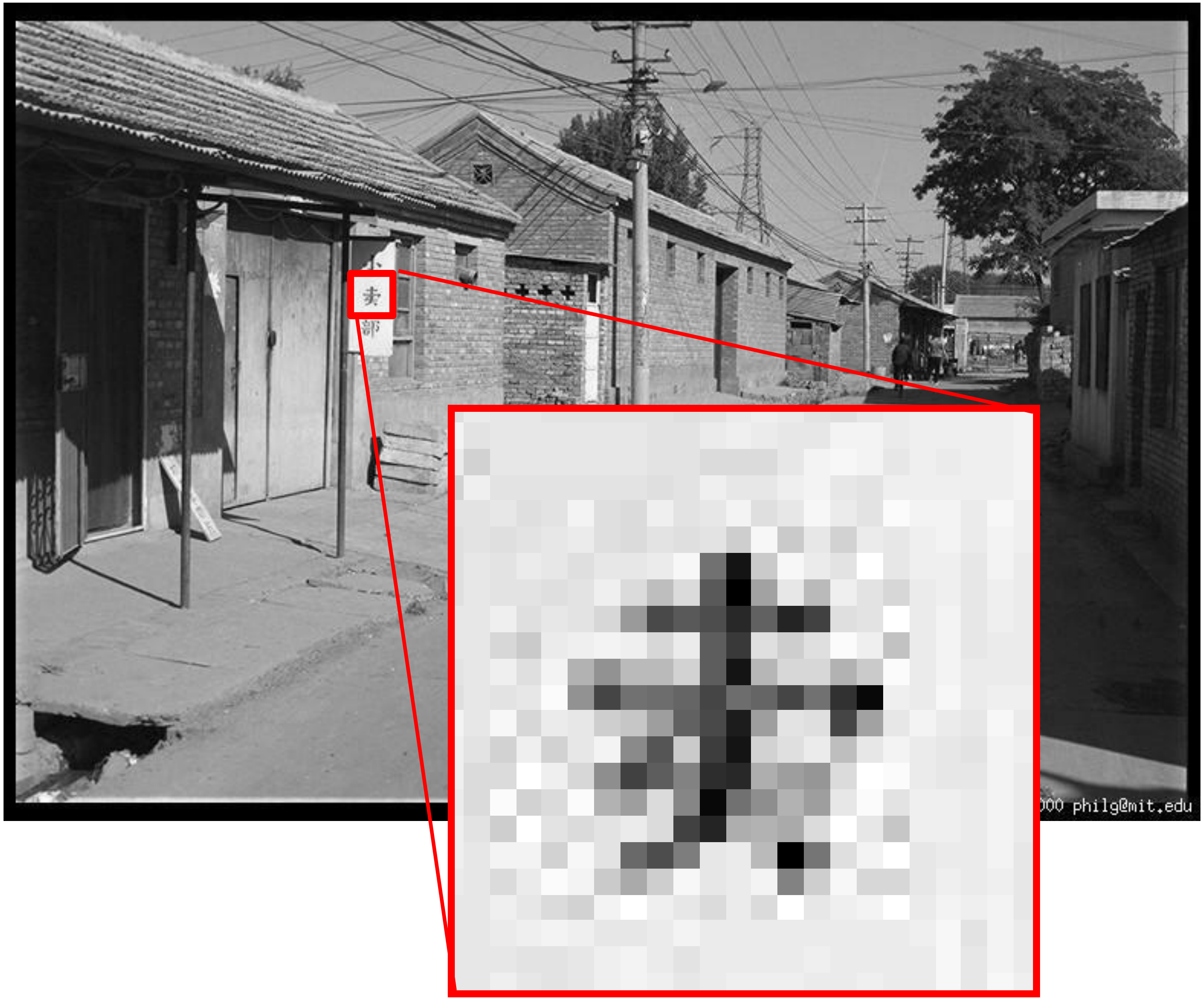
Spatial Resolution

Spatial Resolution

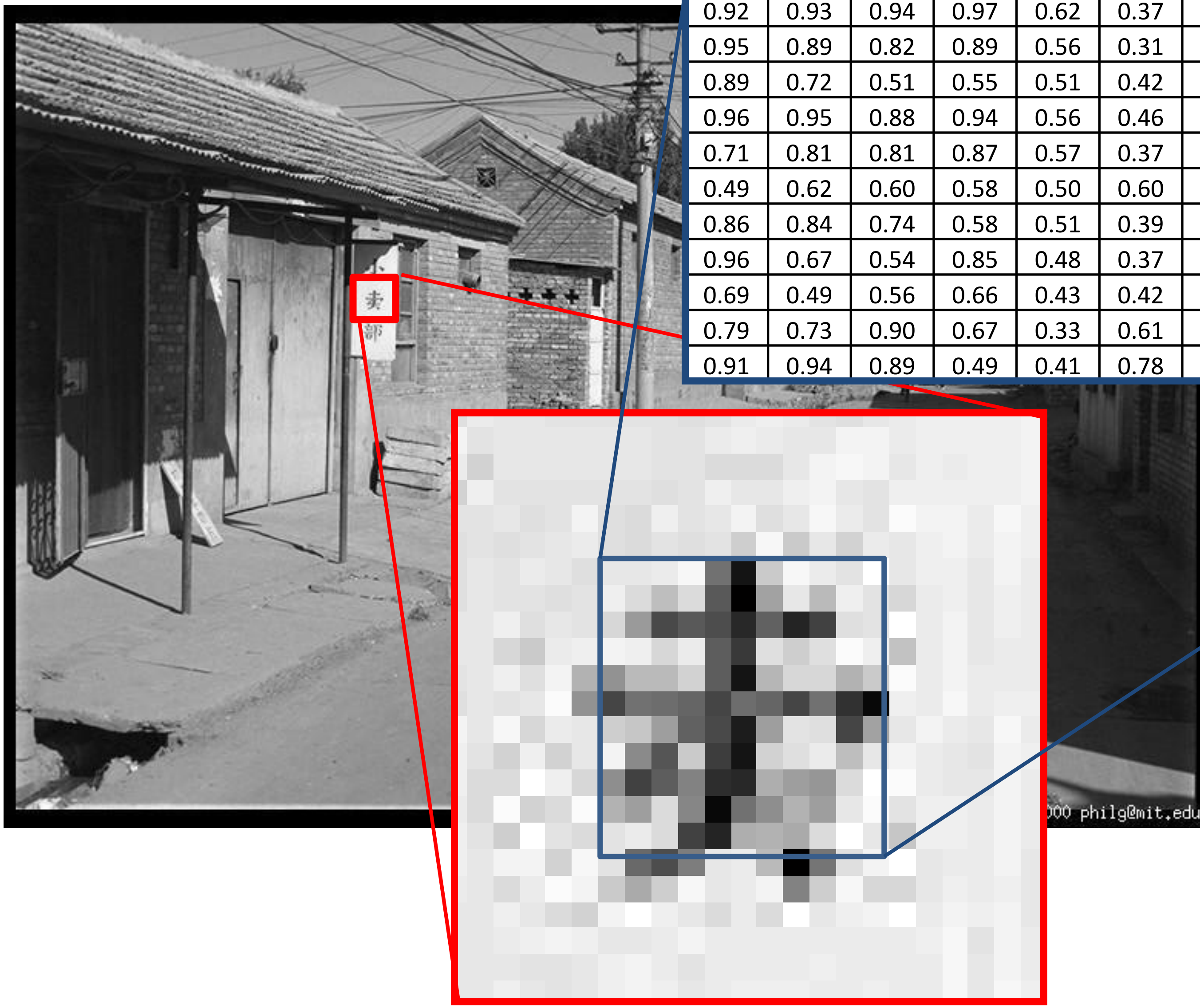
a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

The raster image (pixel matrix)

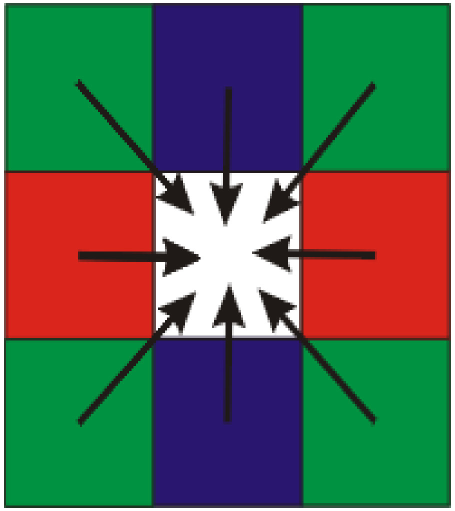


The raster image (pixel matrix)



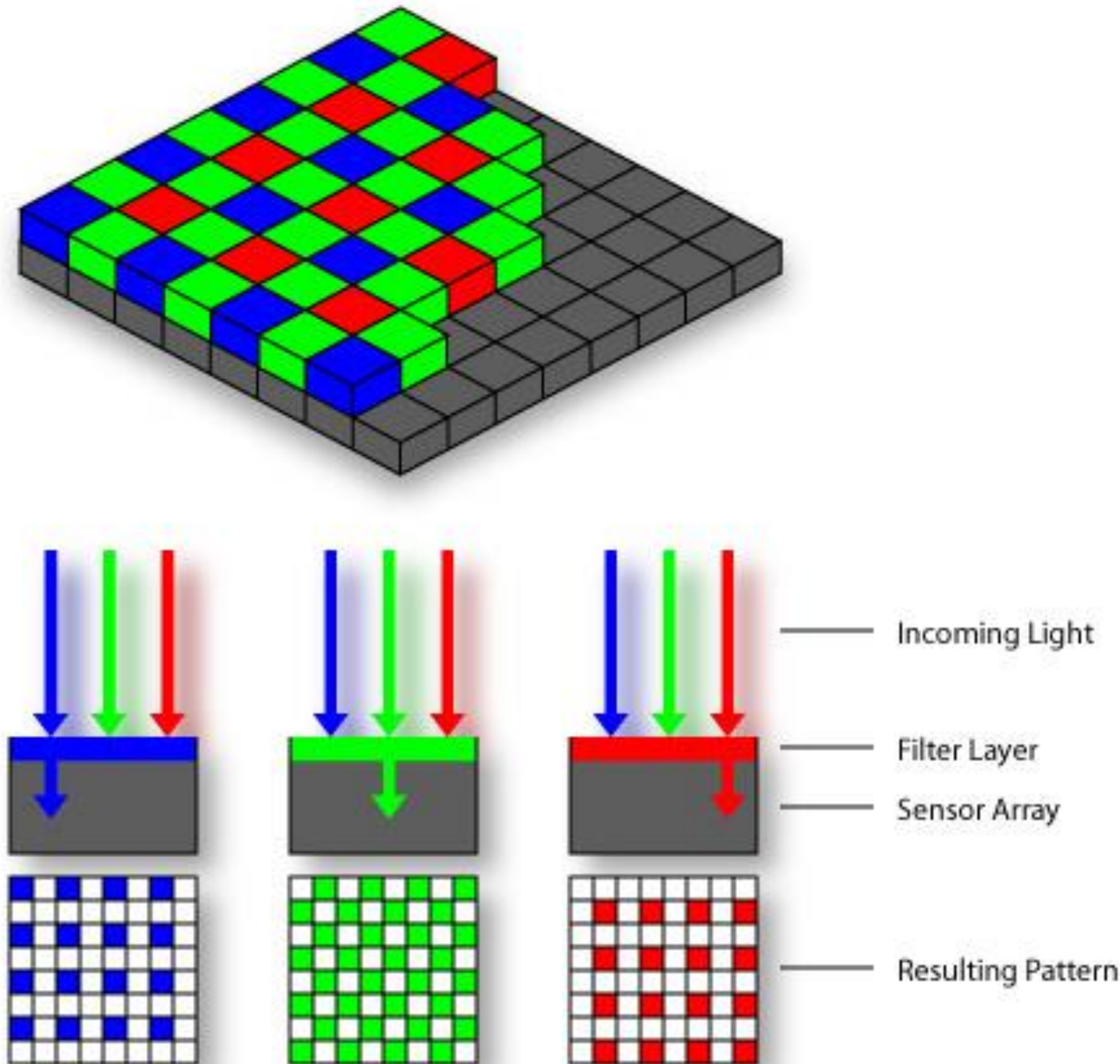
0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

Color Images: Bayer Grid



Estimate RGB at 'G' cells from neighboring values

http://www.cooldictionary.com/words/Bayer-filter.wikipedia

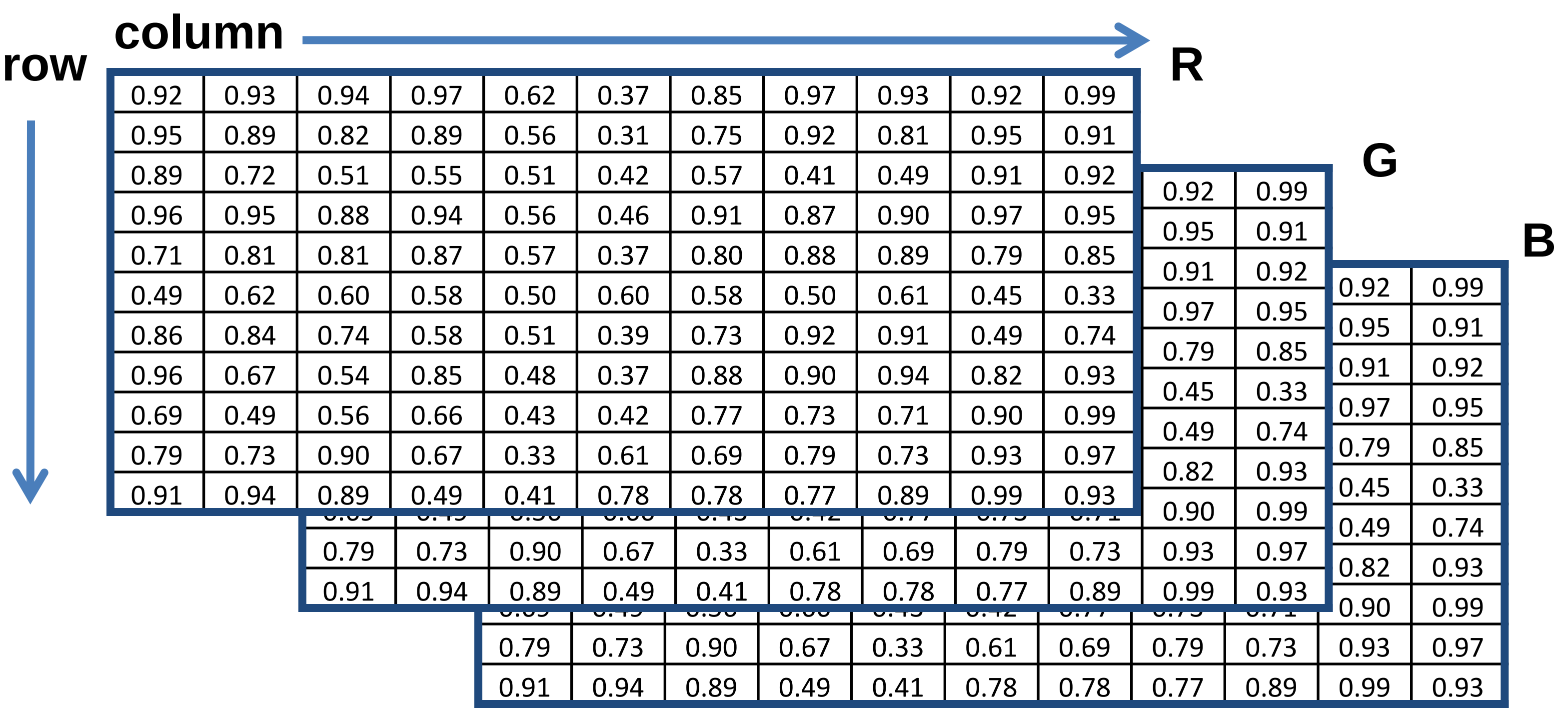


Slide by Steve Seitz

Color Image

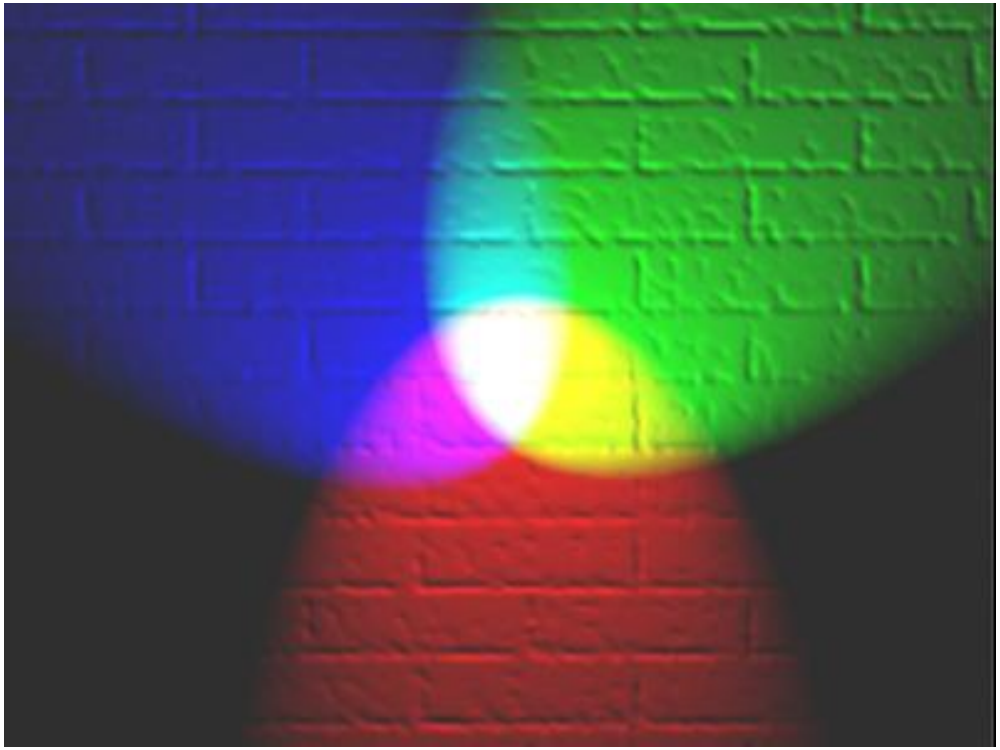
Images in Matlab

- Images represented as a matrix
- Suppose we have a NxM RGB image called "im"
  - im(1,1,1) = top-left pixel value in R-channel
  - im(y, x, b) = y pixels down, x pixels to right in the bth channel
  - im(N, M, 3) = bottom-right pixel in B-channel
- imread(filename) returns a uint8 image (values 0 to 255)
  - Convert to double format (values 0 to 1) with im2double



Color spaces

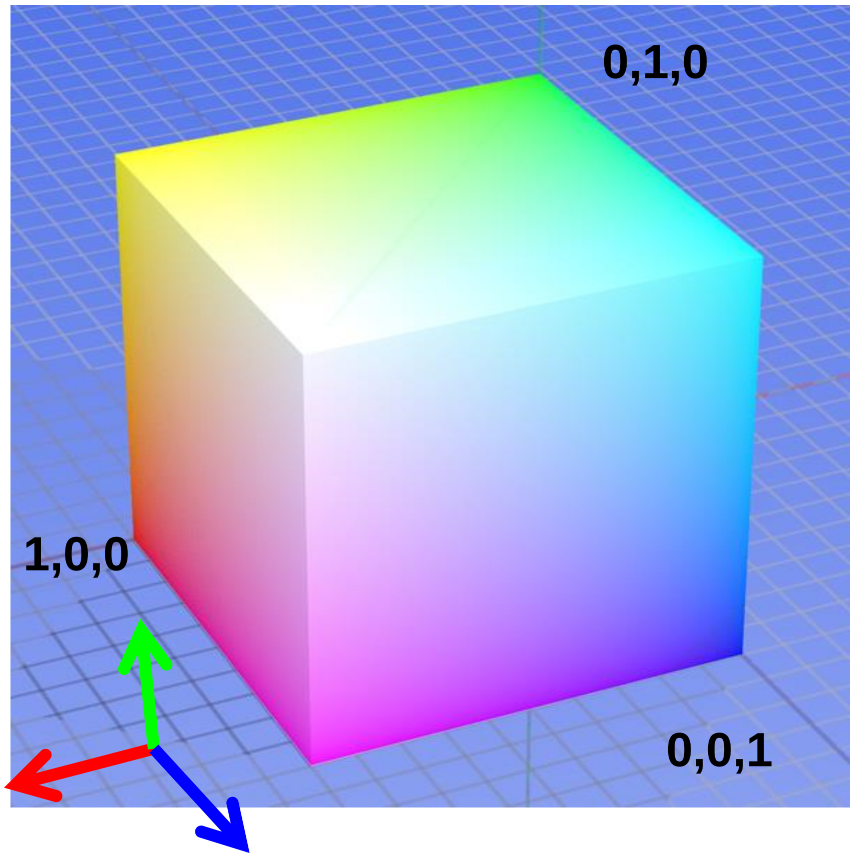
- How can we represent color?



http://en.wikipedia.org/wiki/File:RGB_illumination.jpg

Color spaces: RGB

Default color space



R
(G=0,B=0)

G
(R=0,B=0)

B
(R=0,G=0)

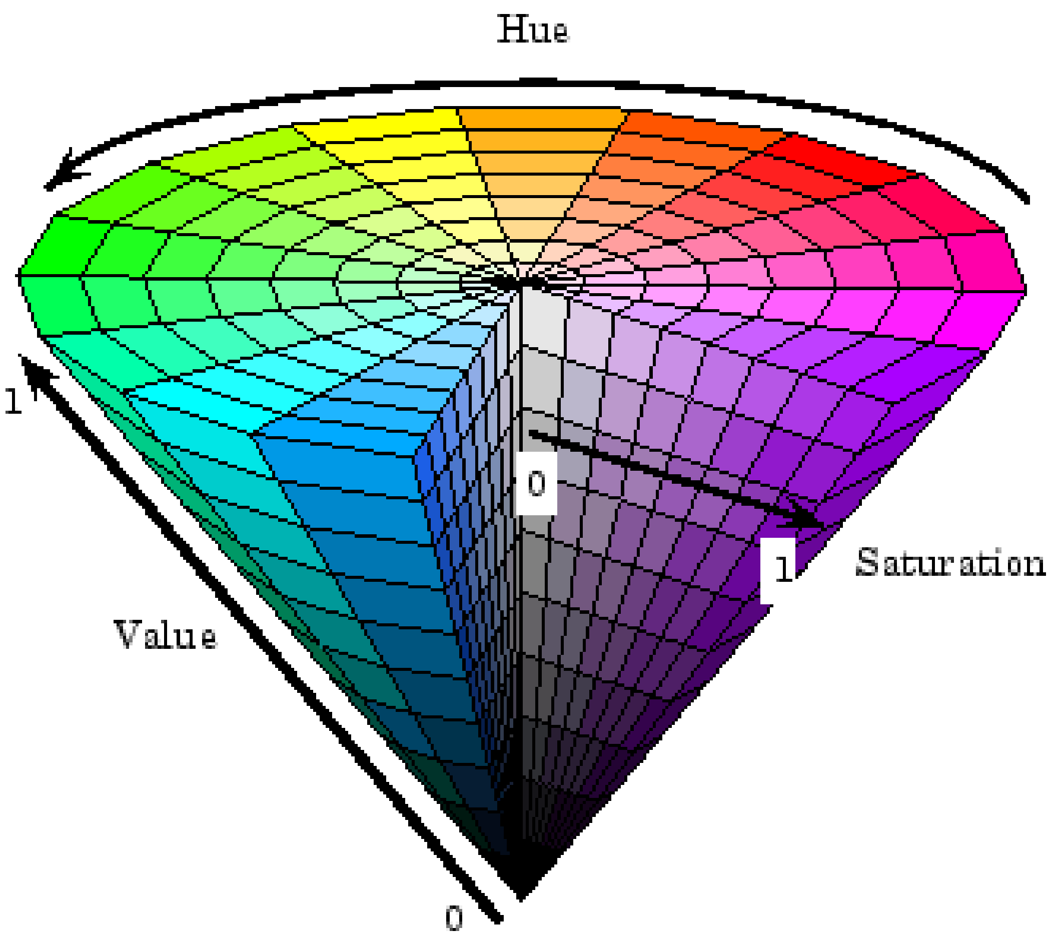
Some drawbacks

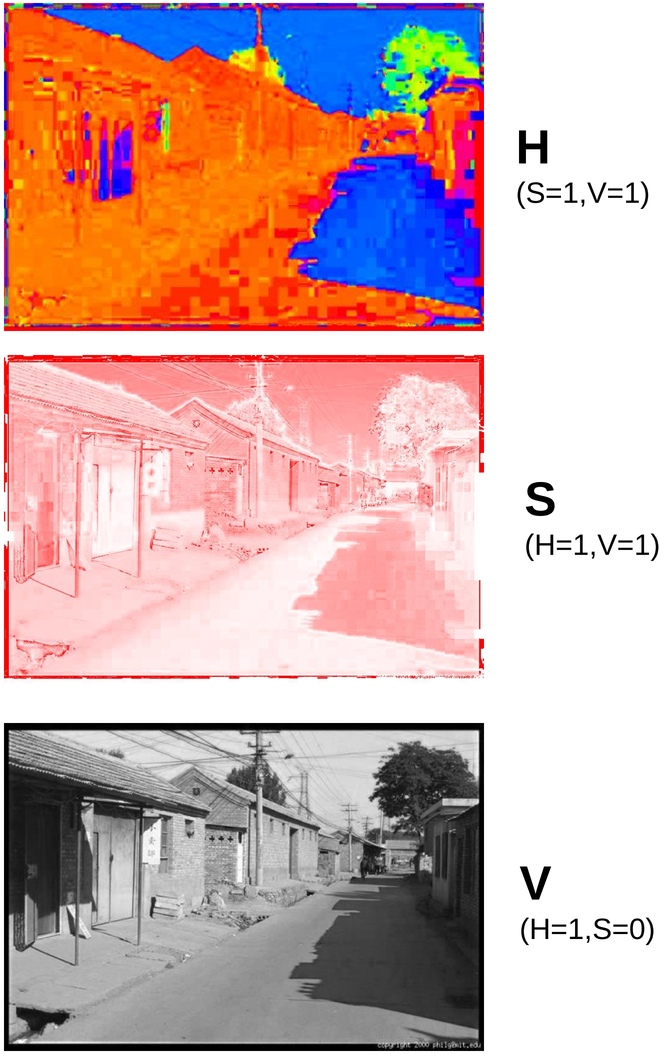
- Strongly correlated channels
- Non-perceptual

Image from: http://en.wikipedia.org/wiki/File:RGB_color_solid_cube.png

Color spaces: HSV

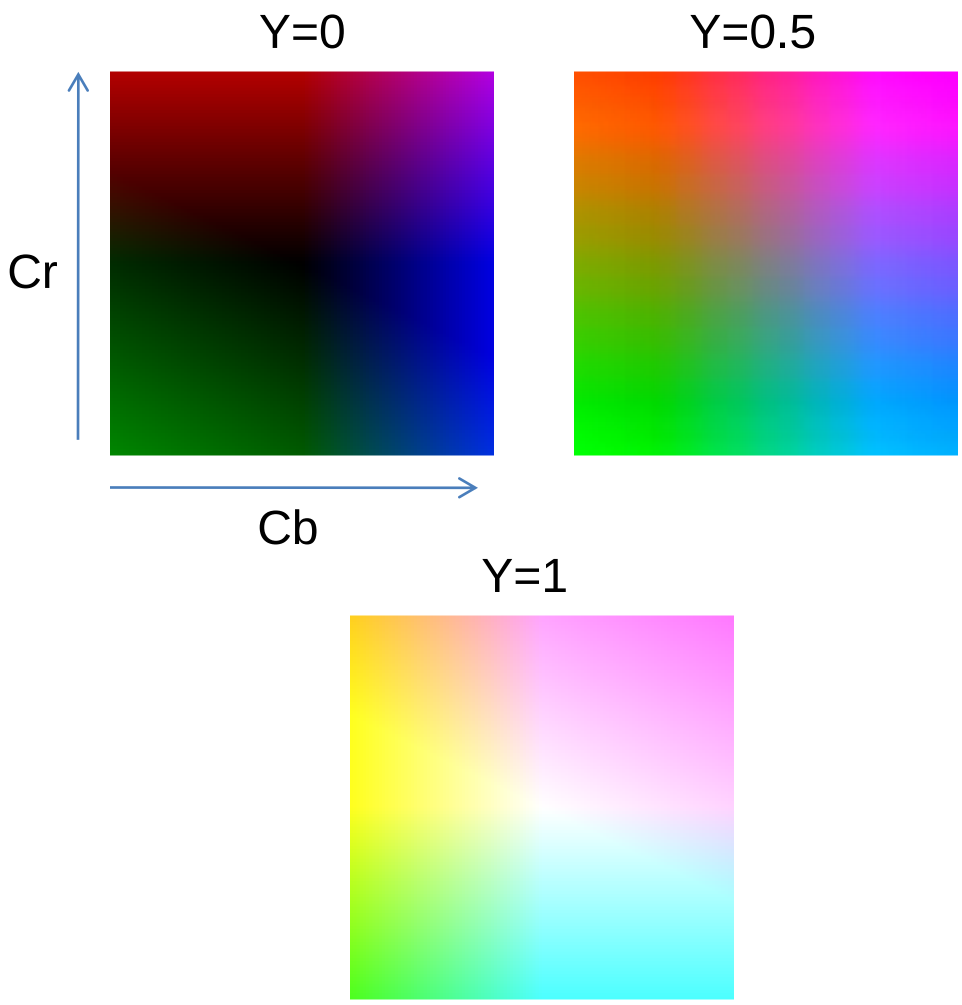
Intuitive color space

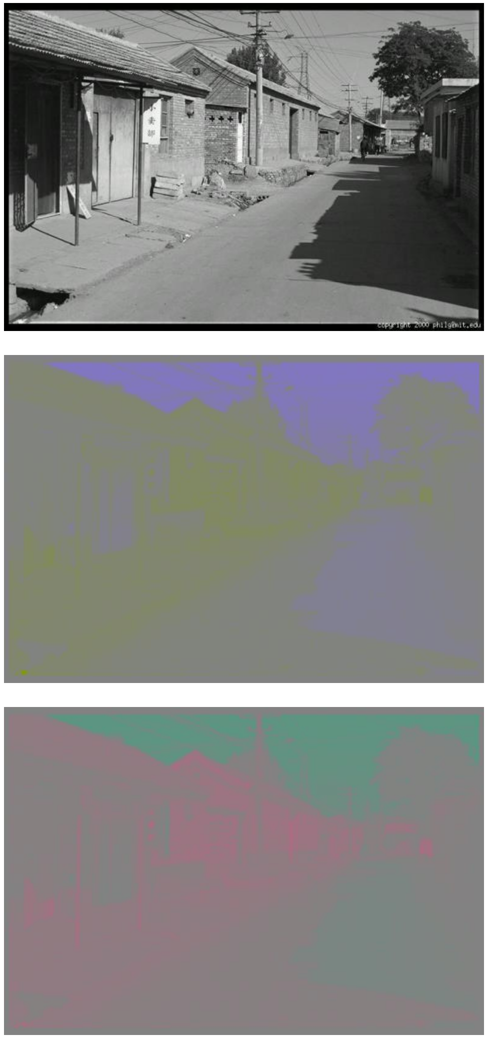




Color spaces: YCbCr

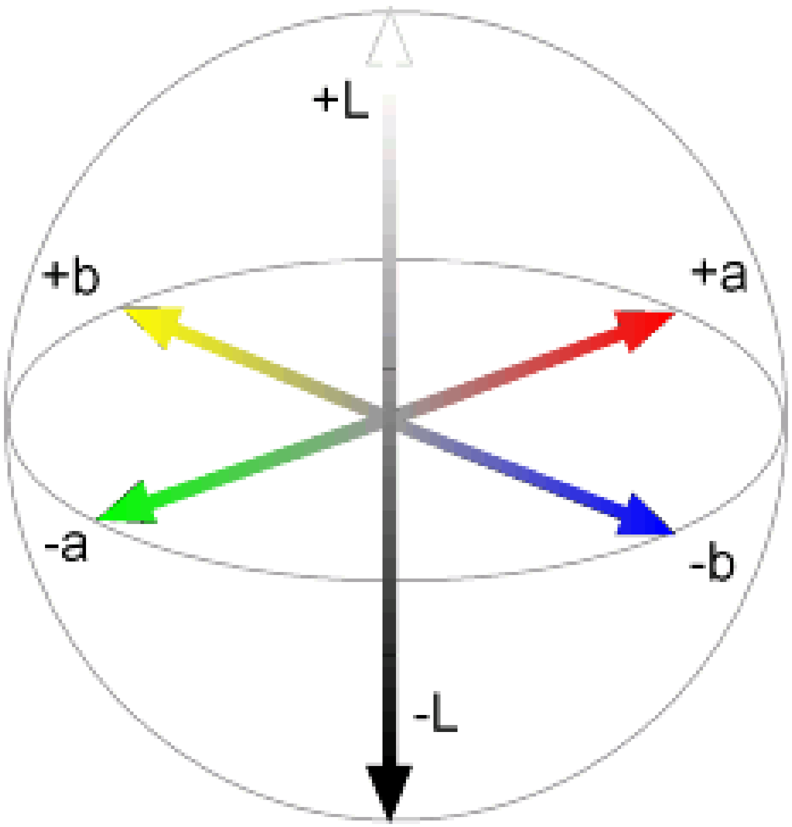
Fast to compute, good for compression, used by TV





Color spaces: L*a*b*

"Perceptually uniform"* color space



L
(a=0,b=0)

a
(L=65,b=0)

b
(L=65,a=0)

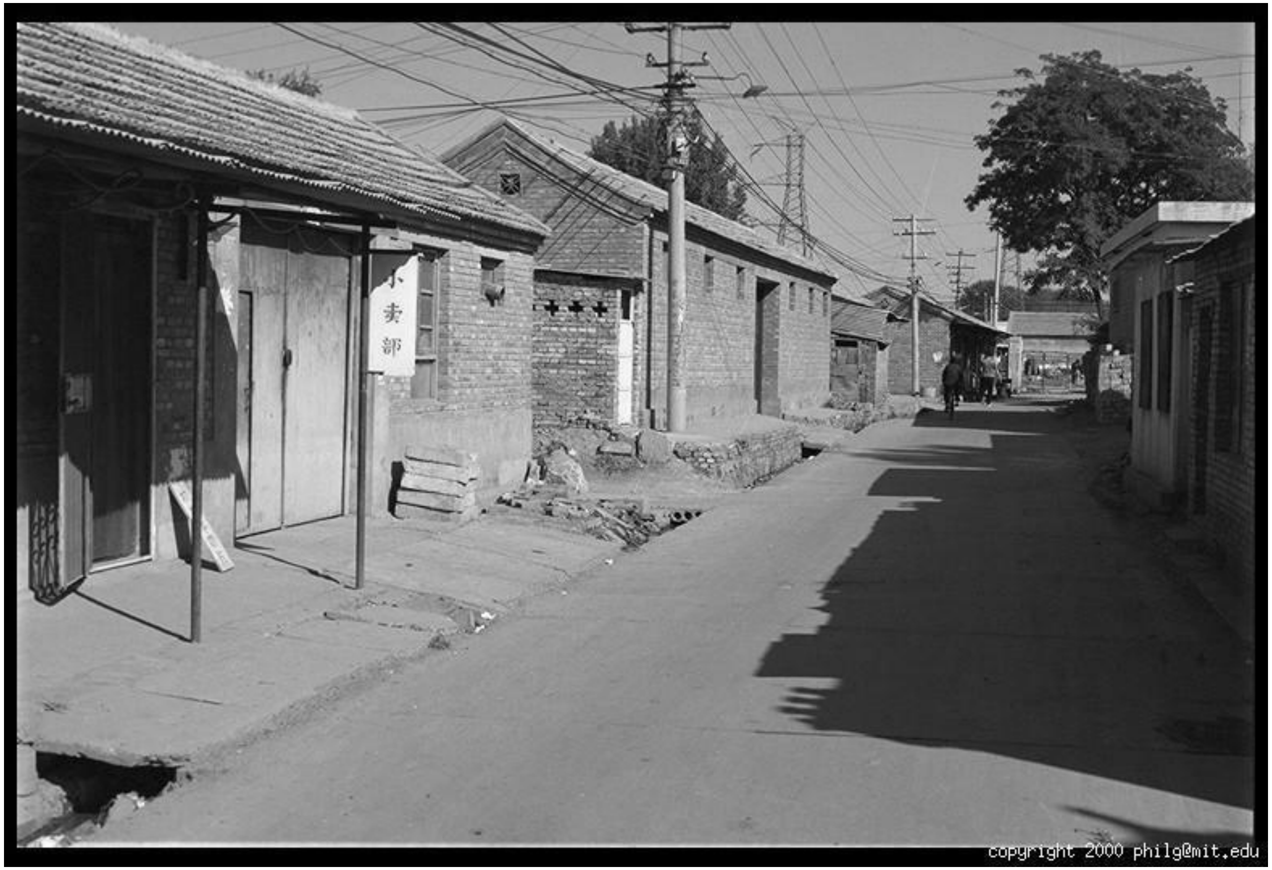
If you had to choose, would you rather go without luminance or chrominance?

If you had to choose, would you rather go without luminance or chrominance?

Most information in intensity

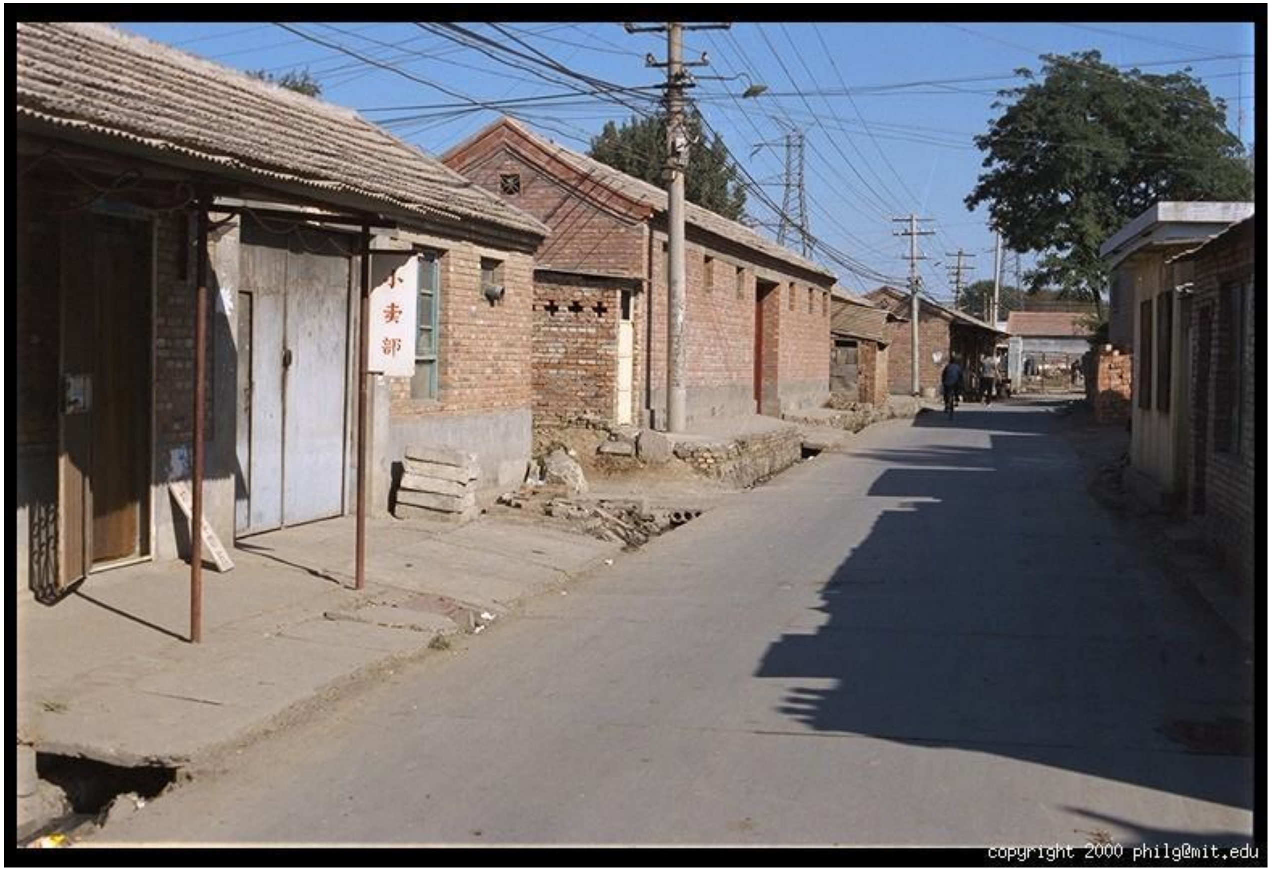
Only color shown – constant intensity

Most information in intensity



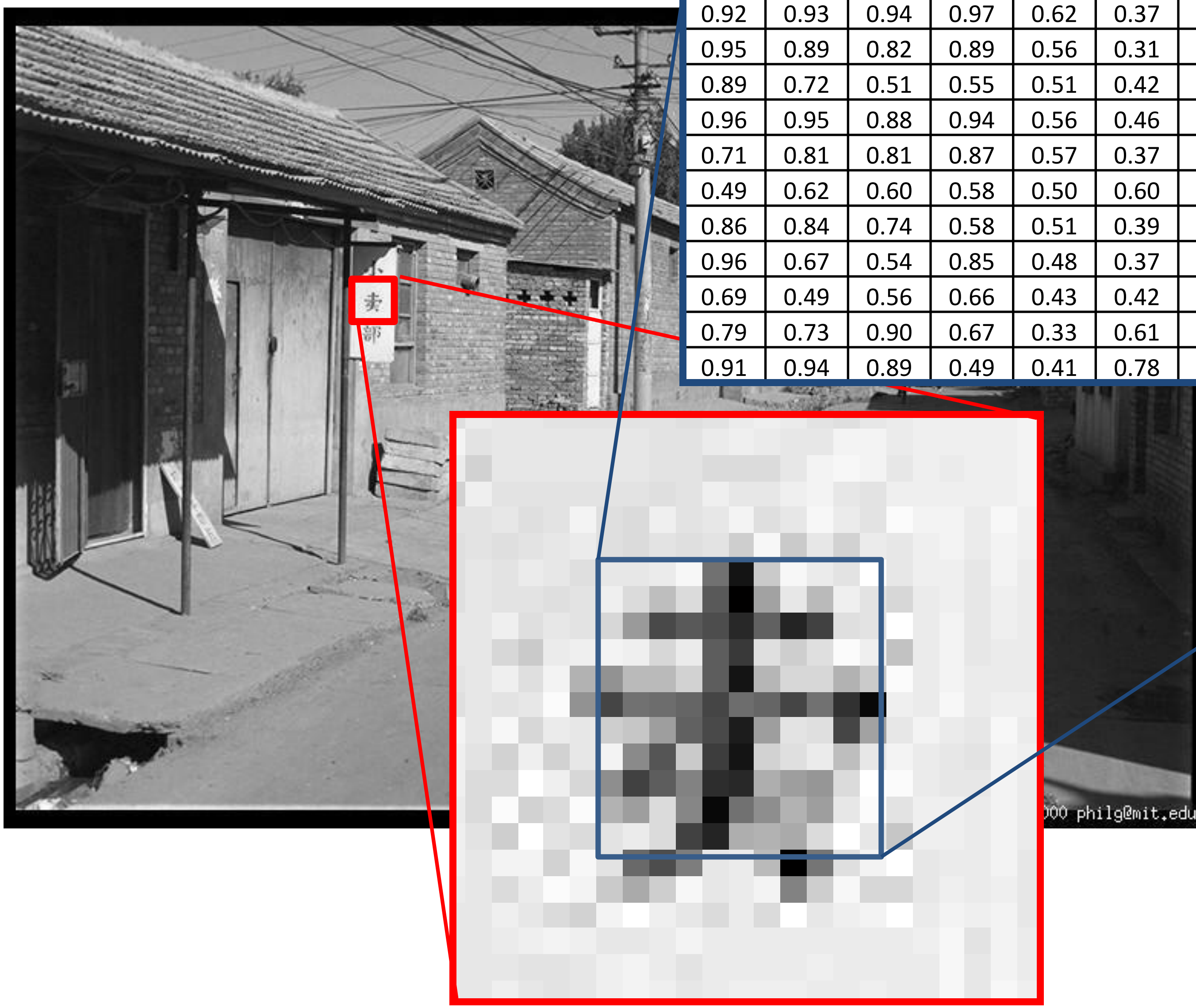
Only intensity shown – constant color

Most information in intensity



Original image

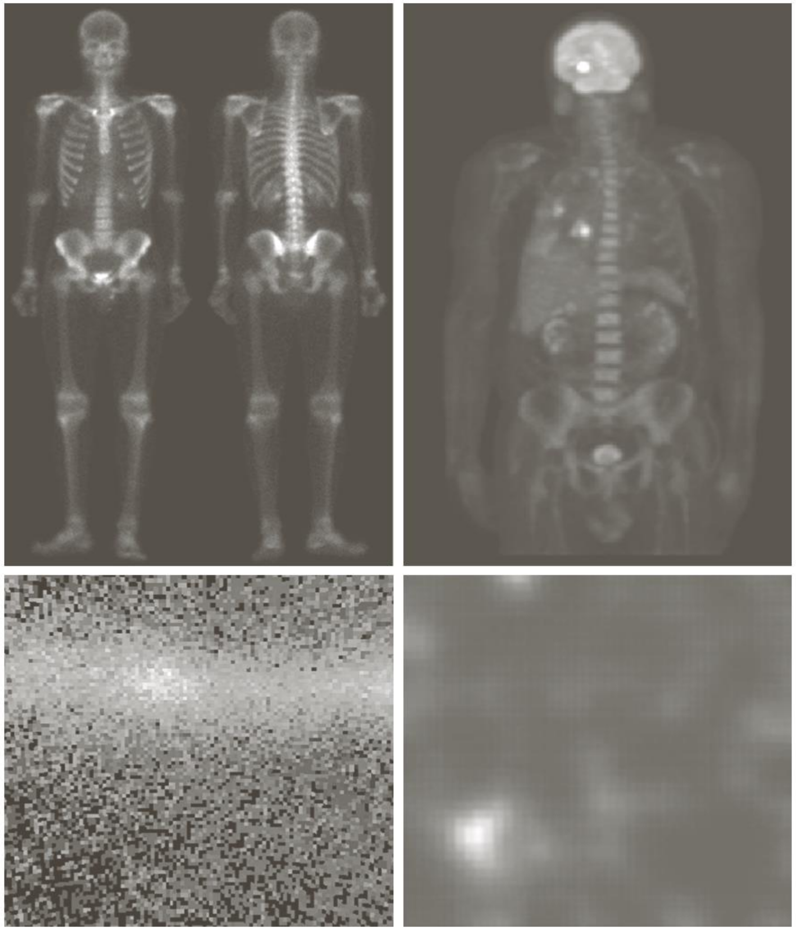
Back to grayscale intensity



0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

Fields that Use Digital Image Processing

- Examples of gamma-ray imaging

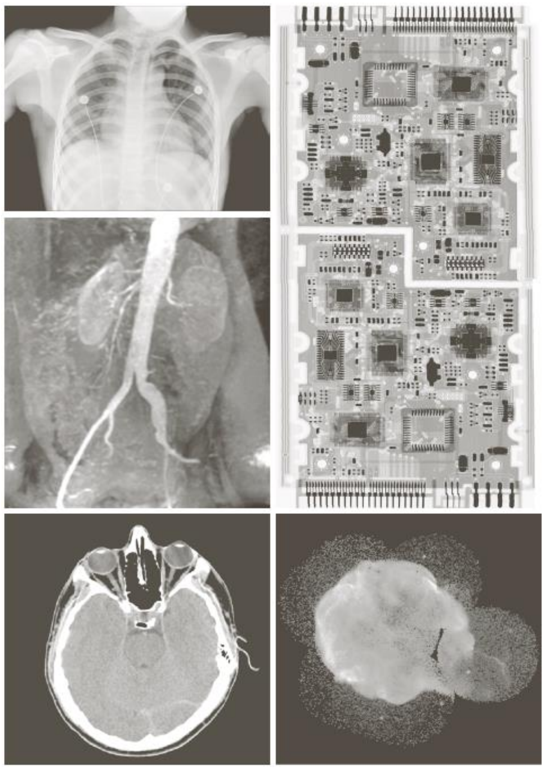


a	b
c	d

FIGURE 1.6 Examples of gamma-ray imaging. (a) Bone scan. (b) PET image. (c) Cygnus Loop. (d) Gamma radiation (bright spot) from a reactor valve. (Images courtesy of (a) G.E. Medical Systems, (b) Dr. Michael E. Casey, CTI PET Systems, (c) NASA, (d) Professors Zhong He and David K. Wehe, University of Michigan.)

Fields that Use Digital Image Processing

- Examples of X-ray imaging

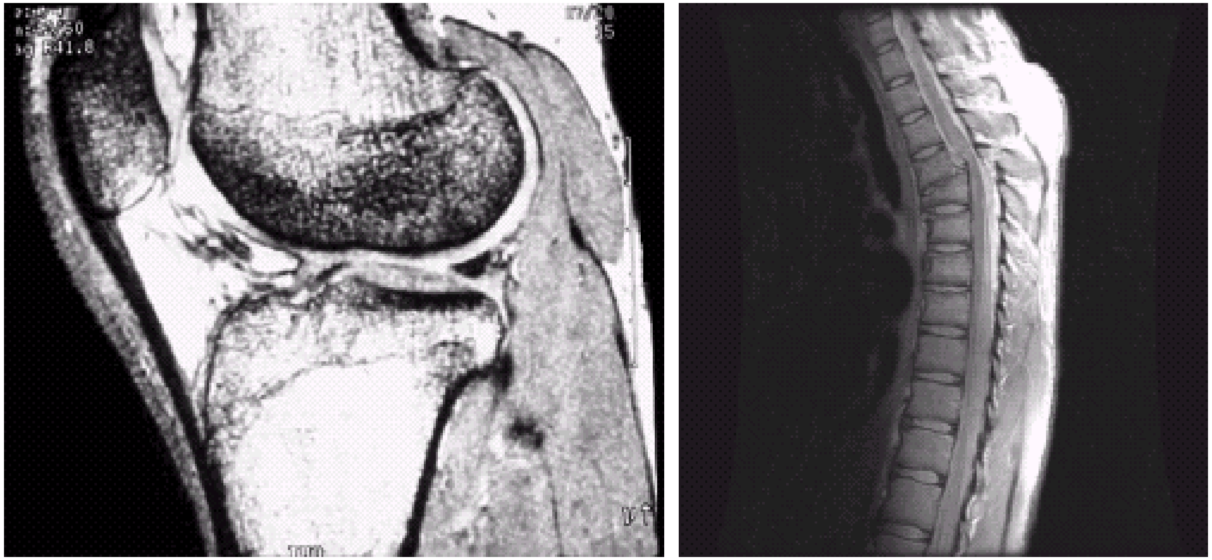


a d
b
c e

FIGURE 1.7 Examples of X-ray imaging. (a) Chest X-ray. (b) Aortic angiogram. (c) Head CT. (d) Circuit boards. (e) Cygnus Loop. (Images courtesy of (a) and (c) Dr. David R. Pickens, Dept. of Radiology & Radiological Sciences, Vanderbilt University Medical Center; (b) Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School; (d) Mr. Joseph E. Pascente, Lixi, Inc.; and (e) NASA.)

The First X-ray Photo
Wilhelm Röntgen
(1845～1923)

Digital Image Signals



a b

FIGURE 1.17 MRI images of a human (a) knee, and (b) spine. (Image (a) courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School, and (b) Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Video Signals

- Moving images (Video)
  - Movie: 24 frames/second
  - TV: 25 frames/second
  - Gray scale image: $f_k(m, n)$
  - Color image:
    $R_k(m, n)$, $G_k(m, n)$, $B_k(m, n)$

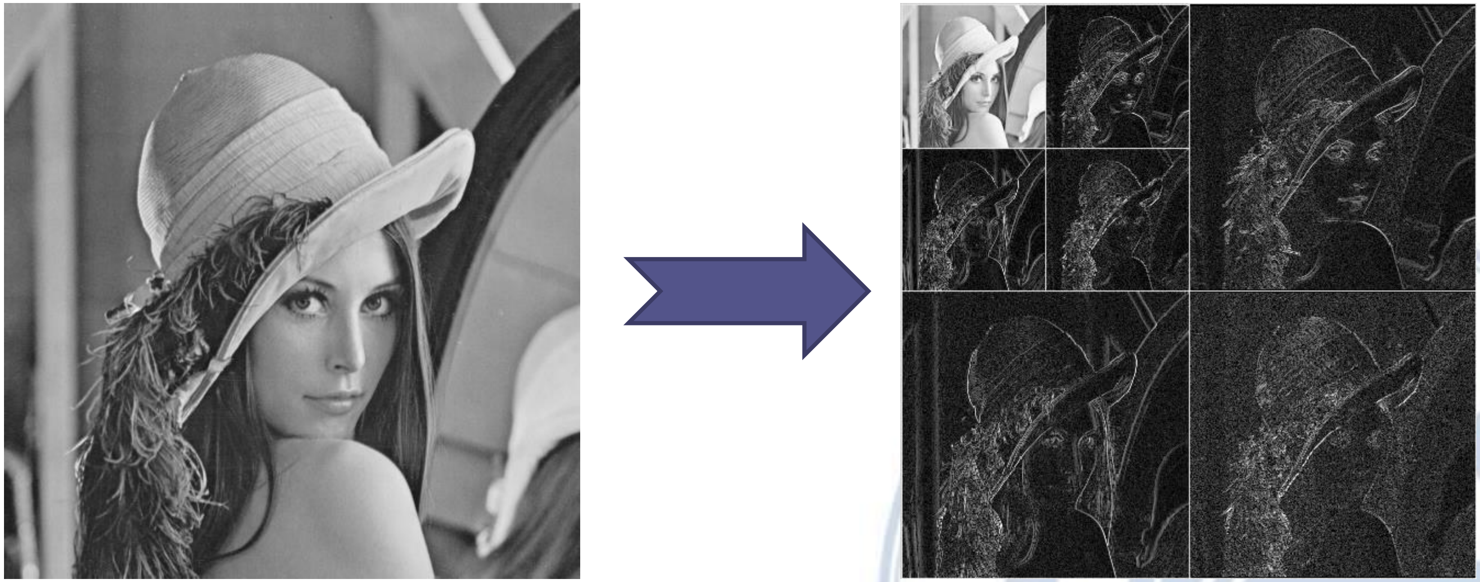
Image Signals and Systems

- Image Compression

Compression at 0.5 bit per pixel by means of JPEG and JPEG2000

Image Signals and Systems

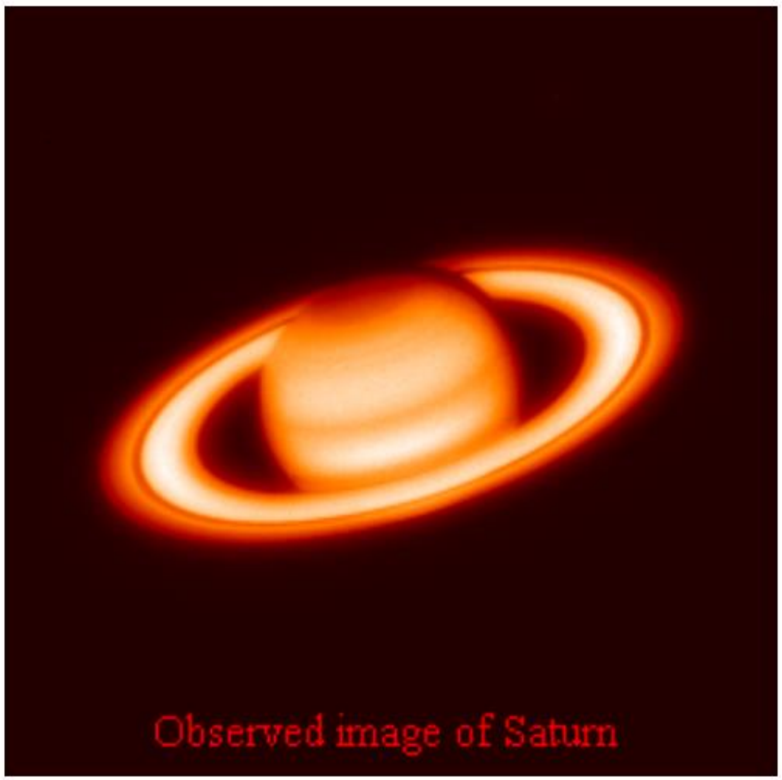
- Image Transform



2-D wavelet transform

Image Signals and Systems

- Restoration of image from Hubble Space Telescope



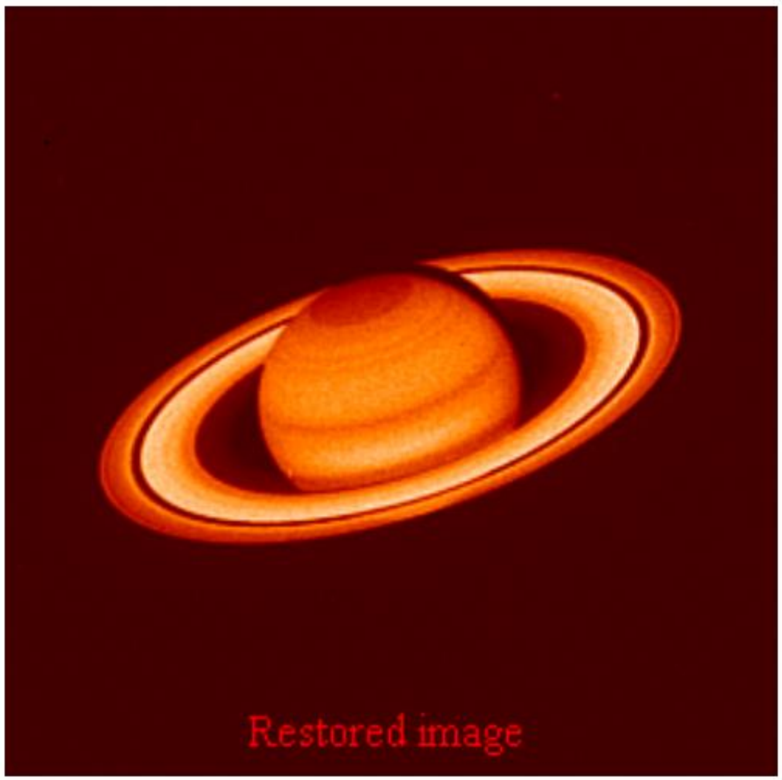
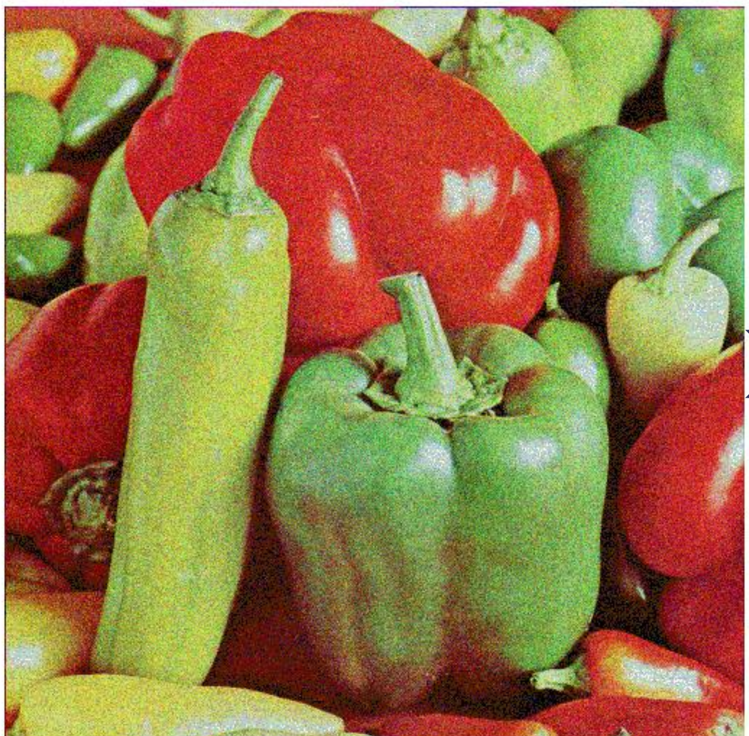


Image Signals and Systems

- Image Denosing

Noisy image$_p$eppers512rgb, PSNR: 20.176 dB (sigma: 25)



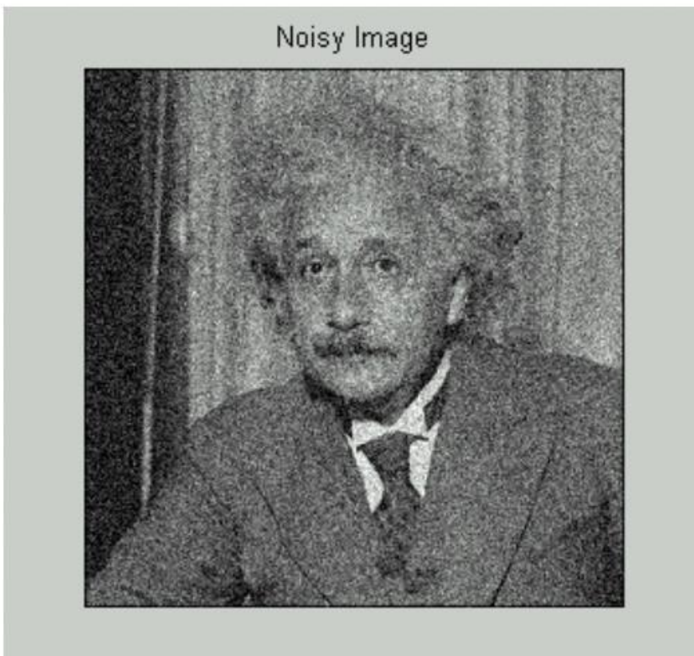
Denoised image$_p$eppers512rgb, PSNR: 31.199 dB

"Image Denoising by Sparse 3D Transform-Domain Collaborative Filtering"

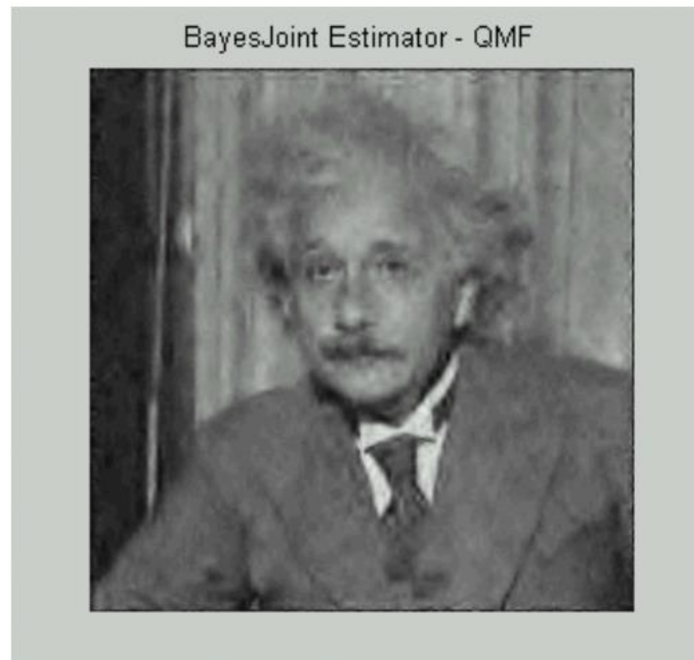
Image Signals and Systems

- I

Noise reduction



Degraded image



Noise-reduced image

Video Signals and Systems

- Video Denosing

“Video Denoising by Sparse 3D Transform-Domain Collaborative Filtering"

Low-level processing

original image

Canny →

edge image

Middle-level processing

edge image

ORT →

data structure

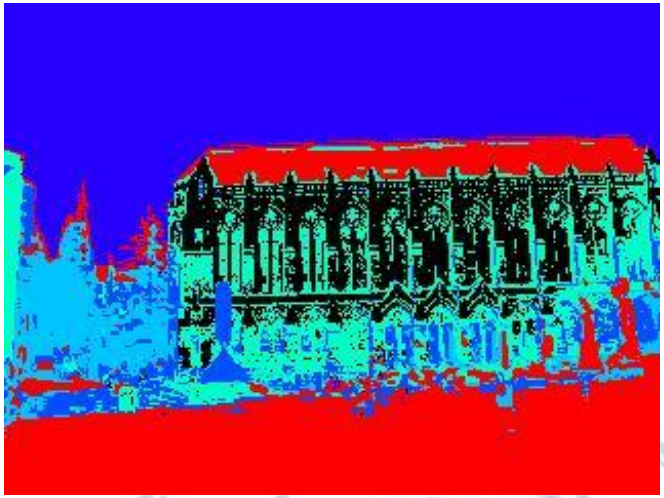
circular arcs and line segments

Middle-level processing

original color image

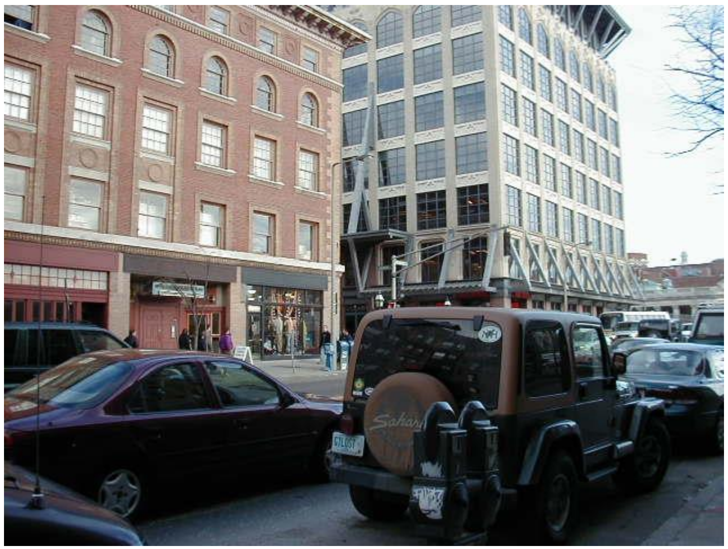
K-means clustering followed by connected component analysis

data structure



regions of homogeneous color

Low-level to high-level processing



low-level →

edge image

↓ middle-level

consistent line clusters

← high-level

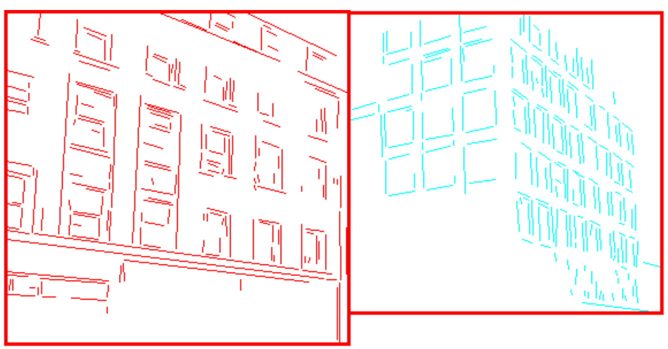
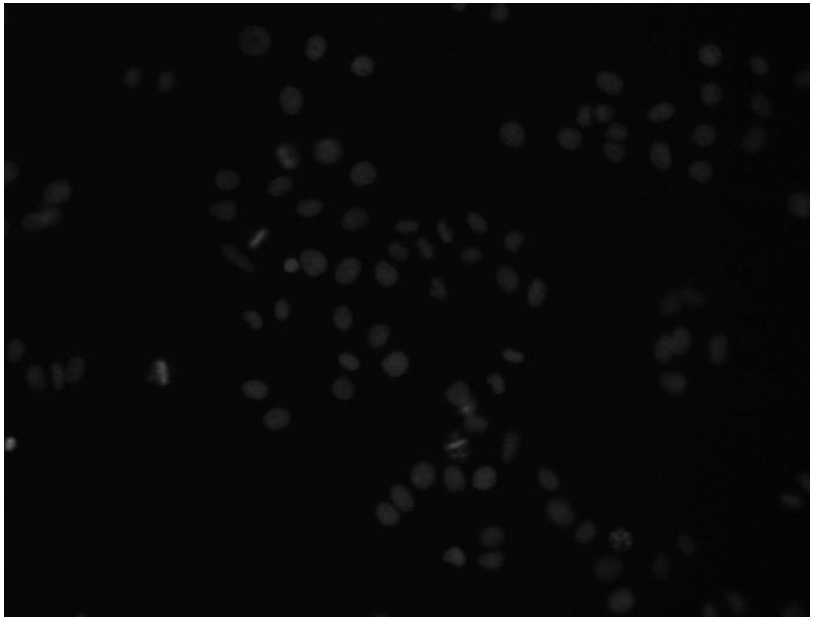


Image Signals and Systems

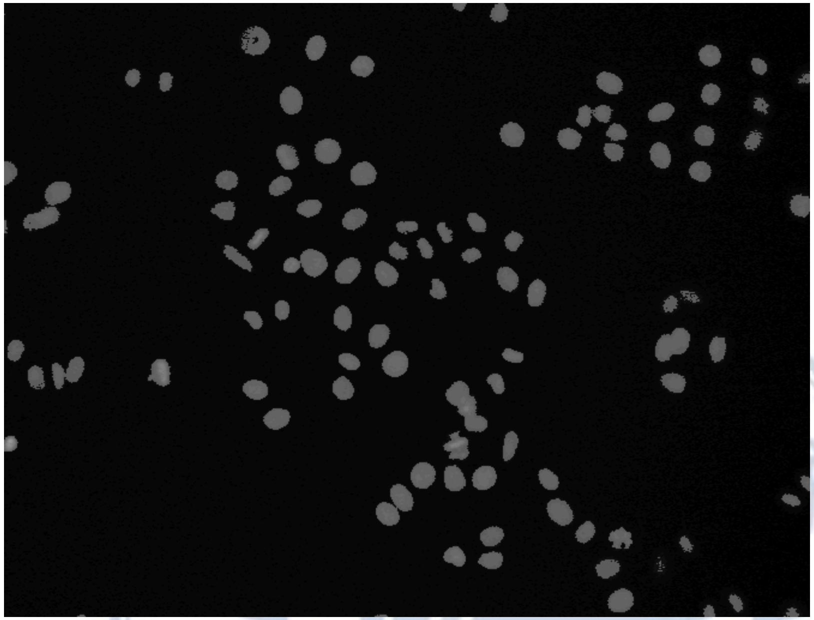
- Middle-level & High-level processing
  - Image → features/attributes, features → recognition
  - Image Analysis, Image Recognition, Image Comprehension
  - Pattern Recognition, Computer Vision
  - Difficulty
    - Computer has no intelligence
    - Machine Learning!!

Image Signals and Systems

- Cell Segmentation (2D)



Original Image



Segment Result

3D Signals and Systems

- Cell Segmentation (3D)

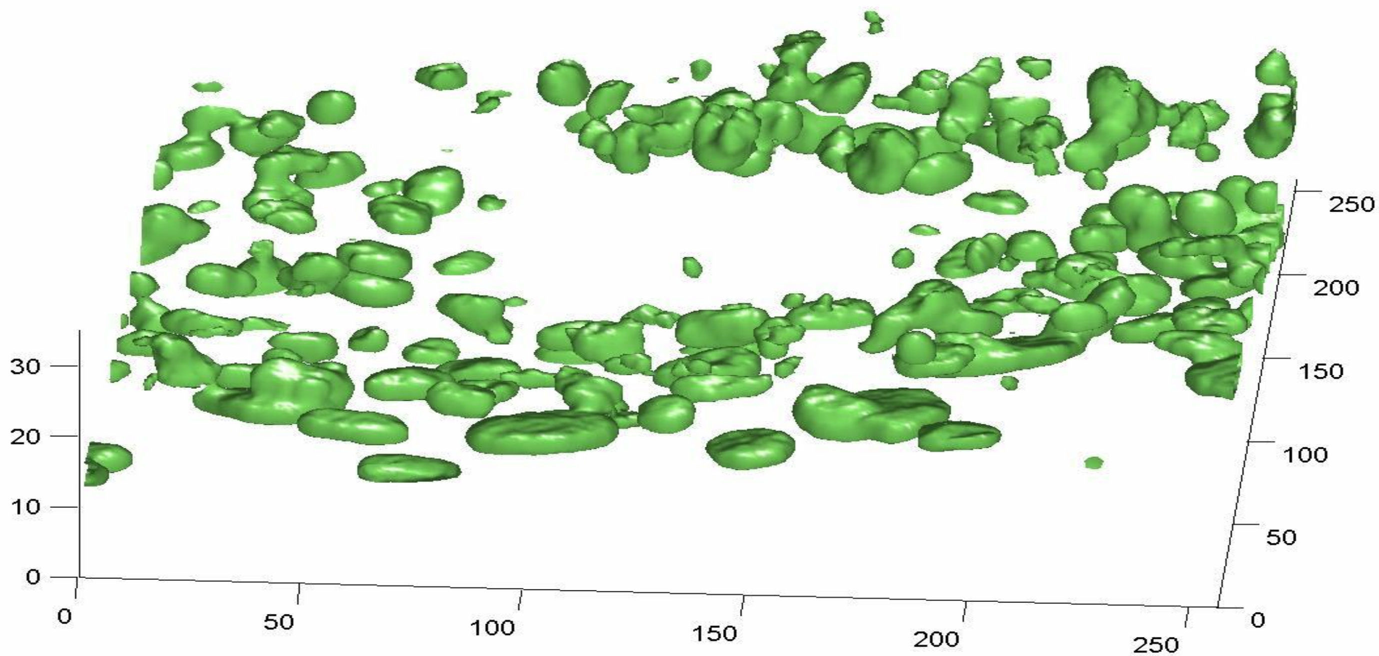
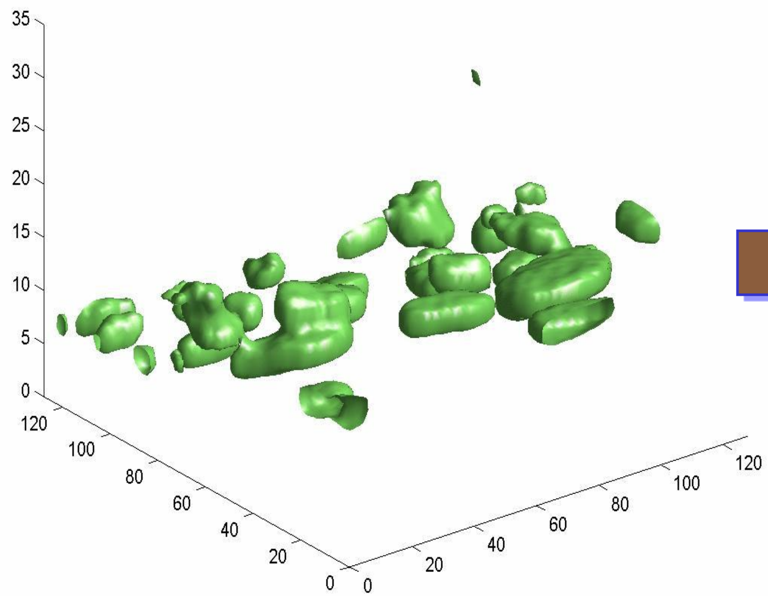


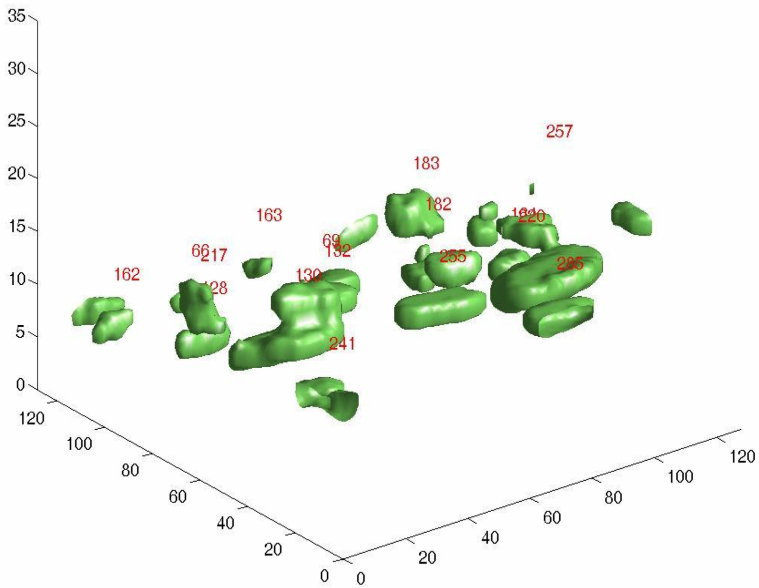
Image Signal and Systems

- Matching Result (2D)

Image Signals and Systems



Segment Result



Matching Result

Image Signals and Systems

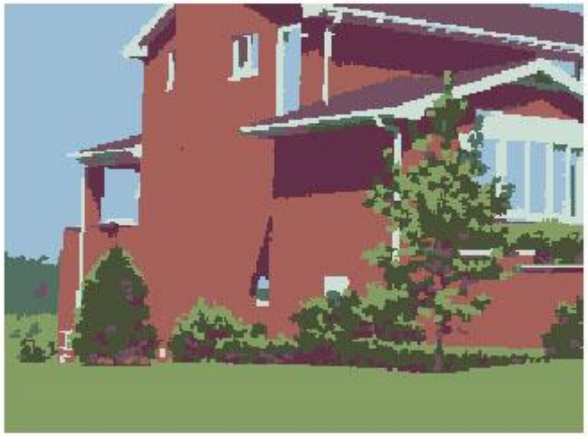
- Edge Detection

gx^2+gy^2

$gx^2+gy^2 > T$

Image Signals and Systems

- Color-Based Segmentation



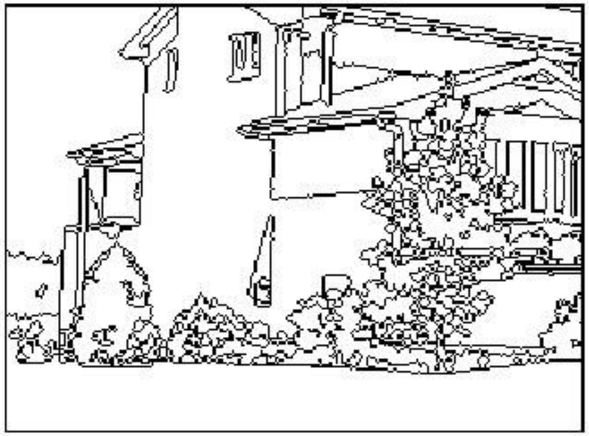
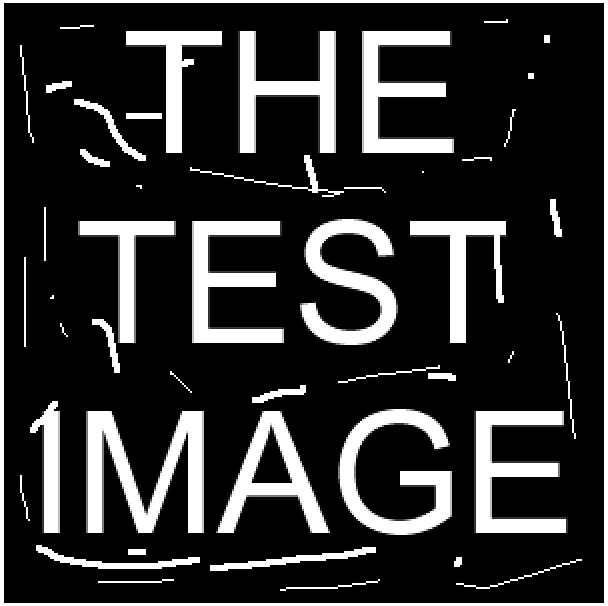
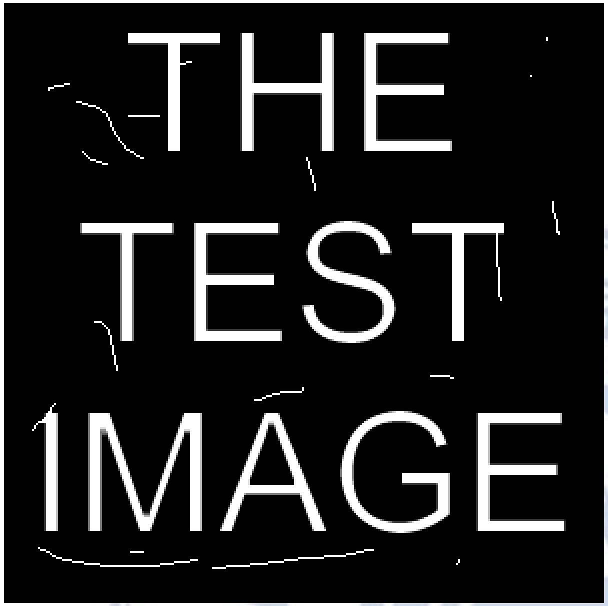


Image Signals and Systems

- Erosion



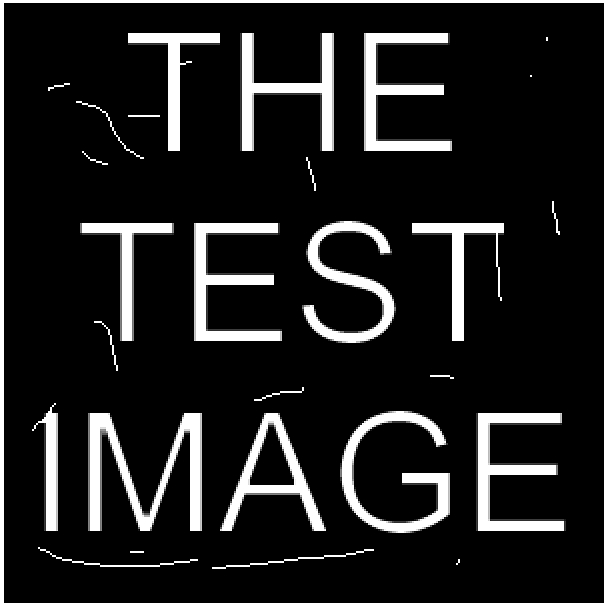
Original image



Eroded image

Image Signals and Systems

- Erosion



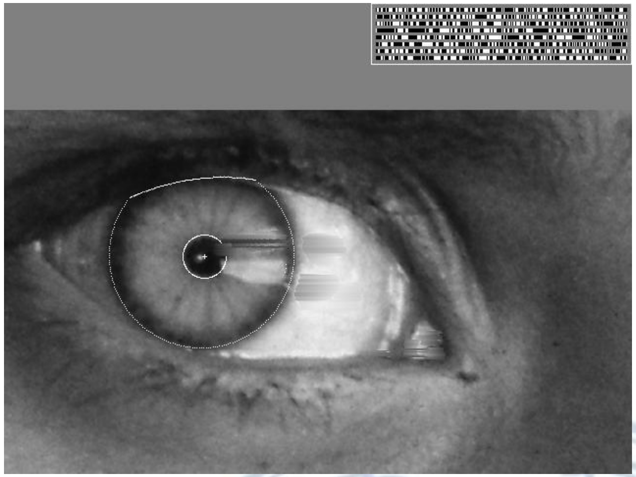
Eroded once

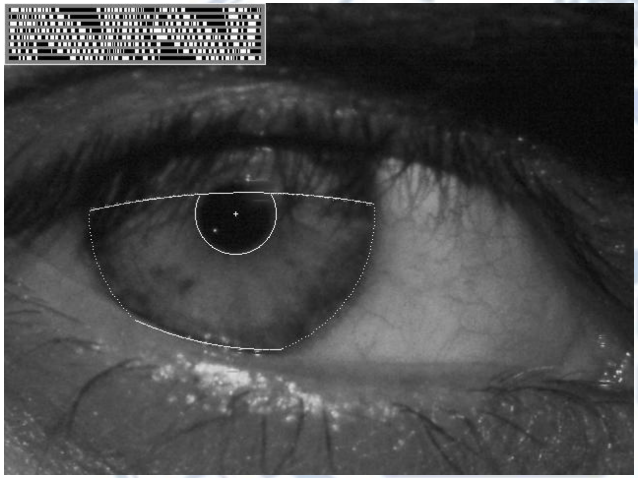
Eroded twice

Vision Signals and Systems

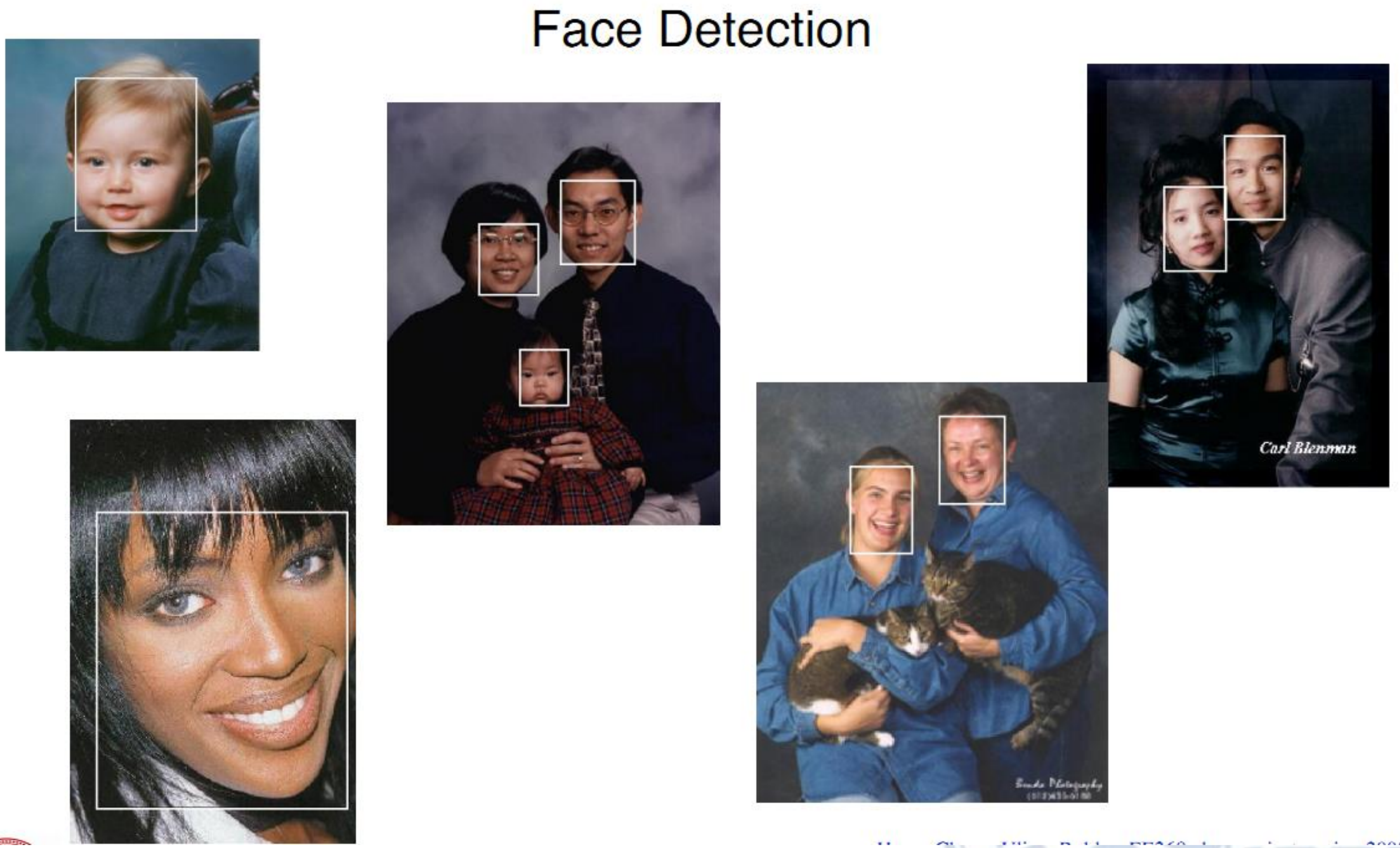
- Vision-based biometrics

The Afghan Girl Identified by Her Iris Patterns





Vision Signals and Systems



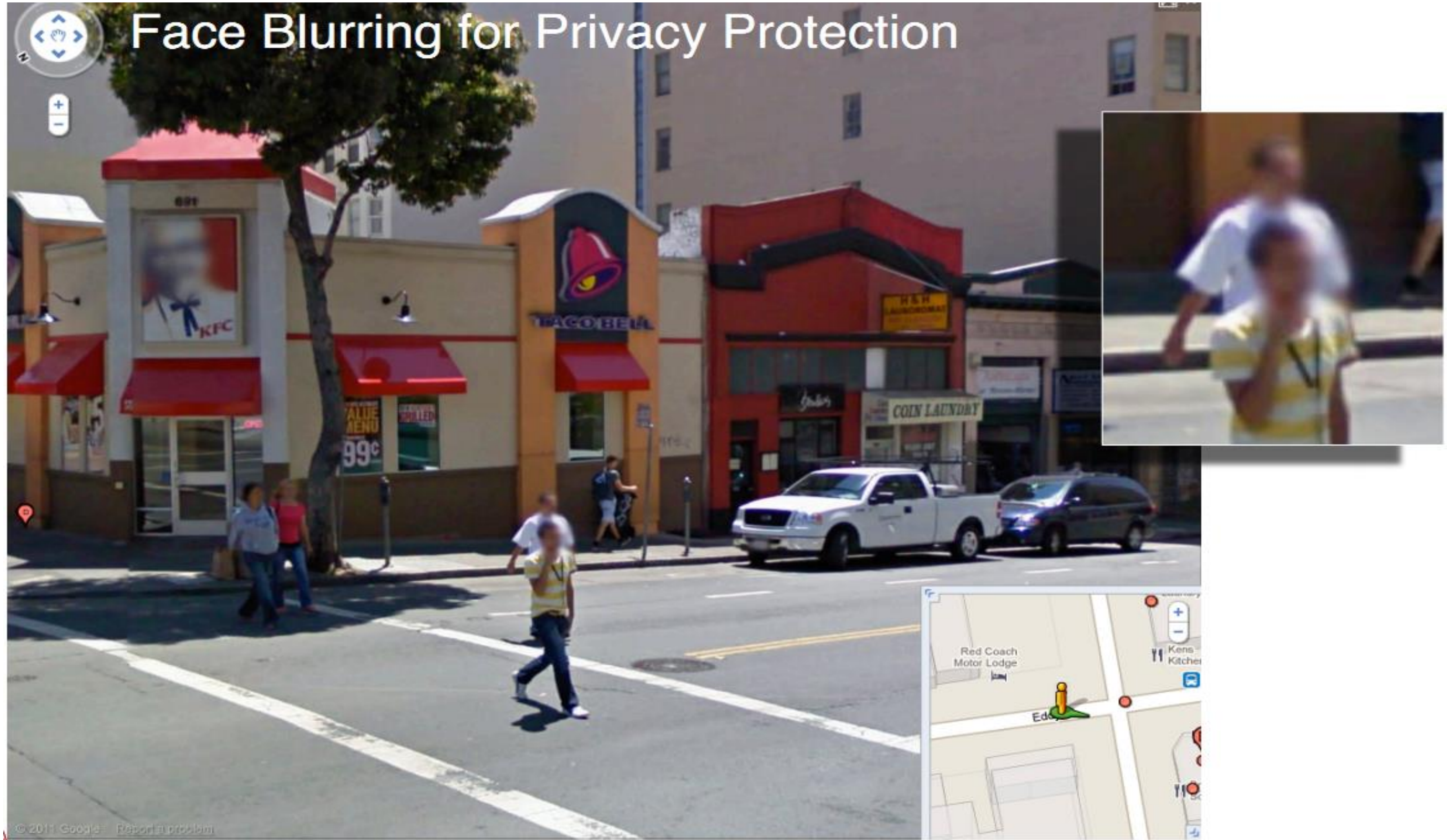
Vision Signals and Systems

Vision Signals and Systems

- Surveillance and tracking

Institute of Media,
Information, and Network

Vision Signals and Systems

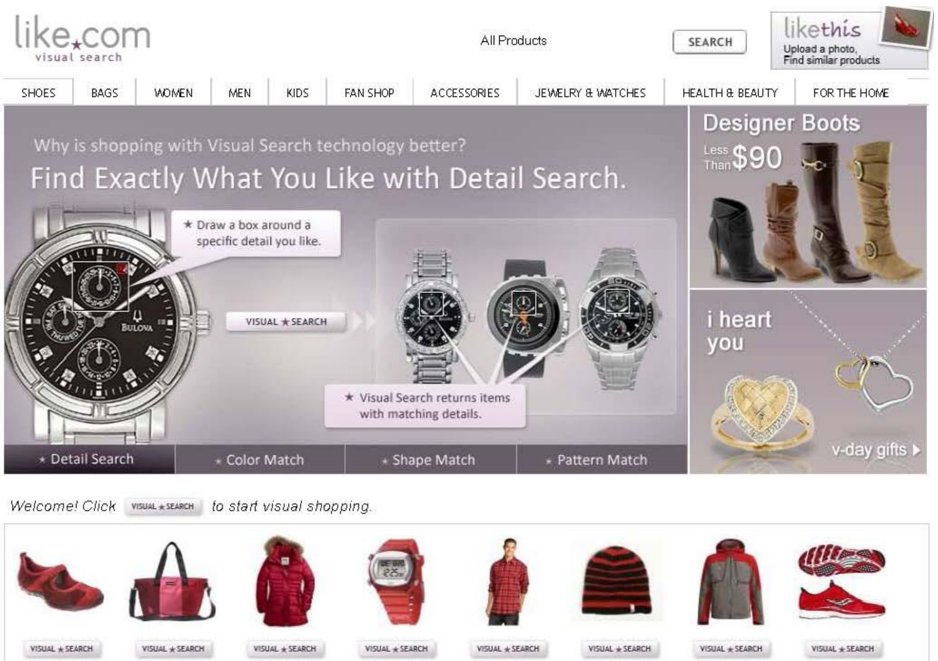


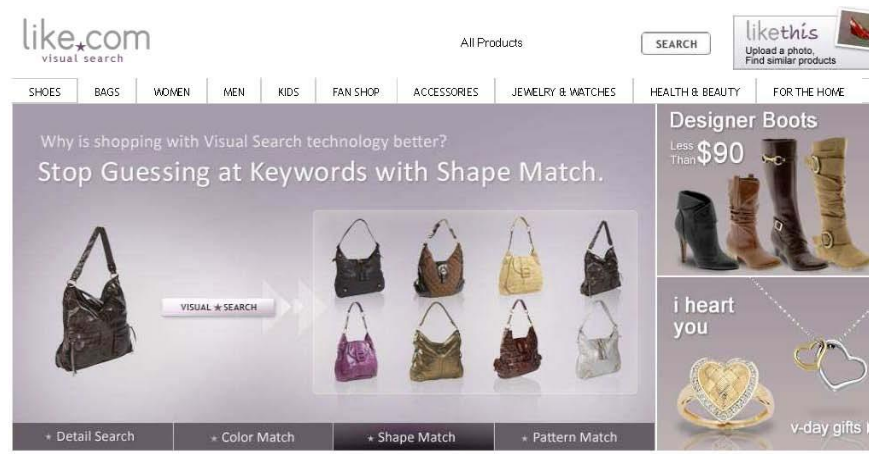
Vision Signals and Systems

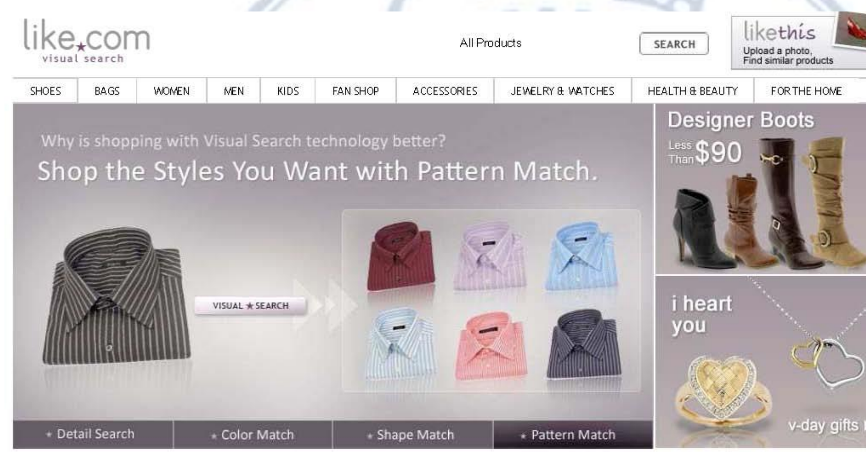
- Augmented reality

Vision Signals and Systems

- Content-based retrieval



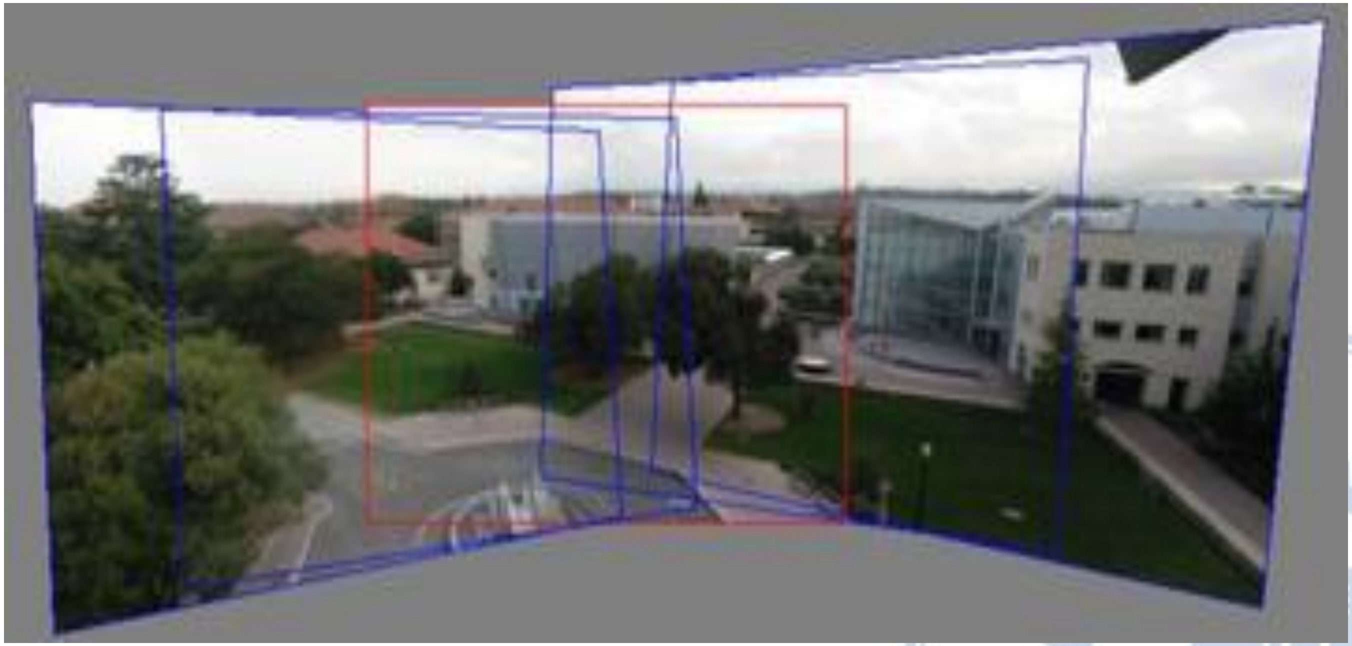




Online shopping catalog search

Something Cool!!!

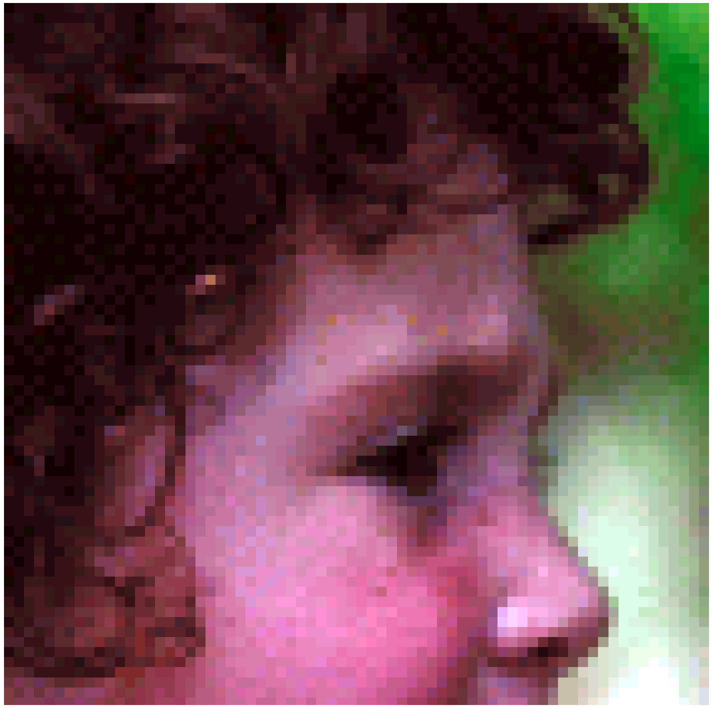
- Panoramas

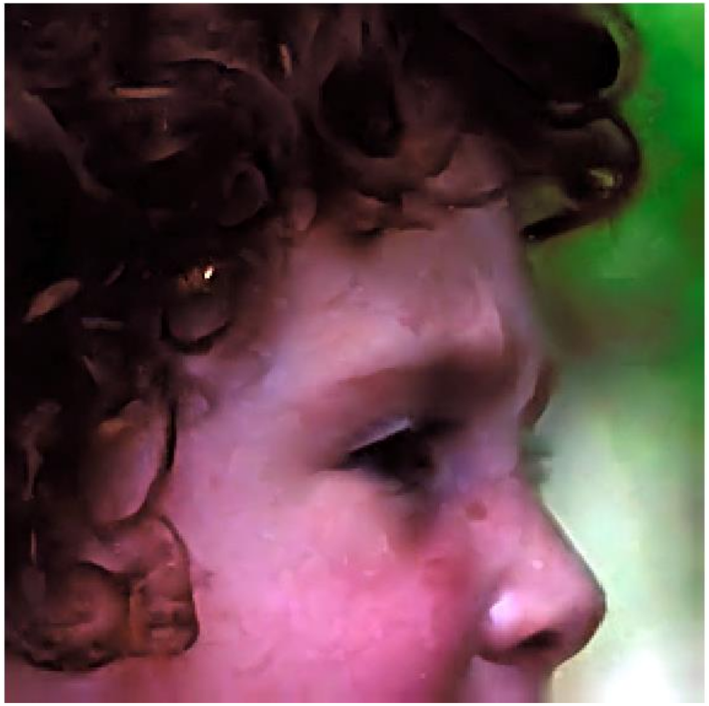


1. Pick one image (red)
2. Warp the other images towards it (usually, one by one)
3. blend

Something Cool!!!

Super-resolution





Something Cool!!!

- Panoramas

- Automatic Mosaic Stitching

Something Cool!!!

- Face warping and morphing

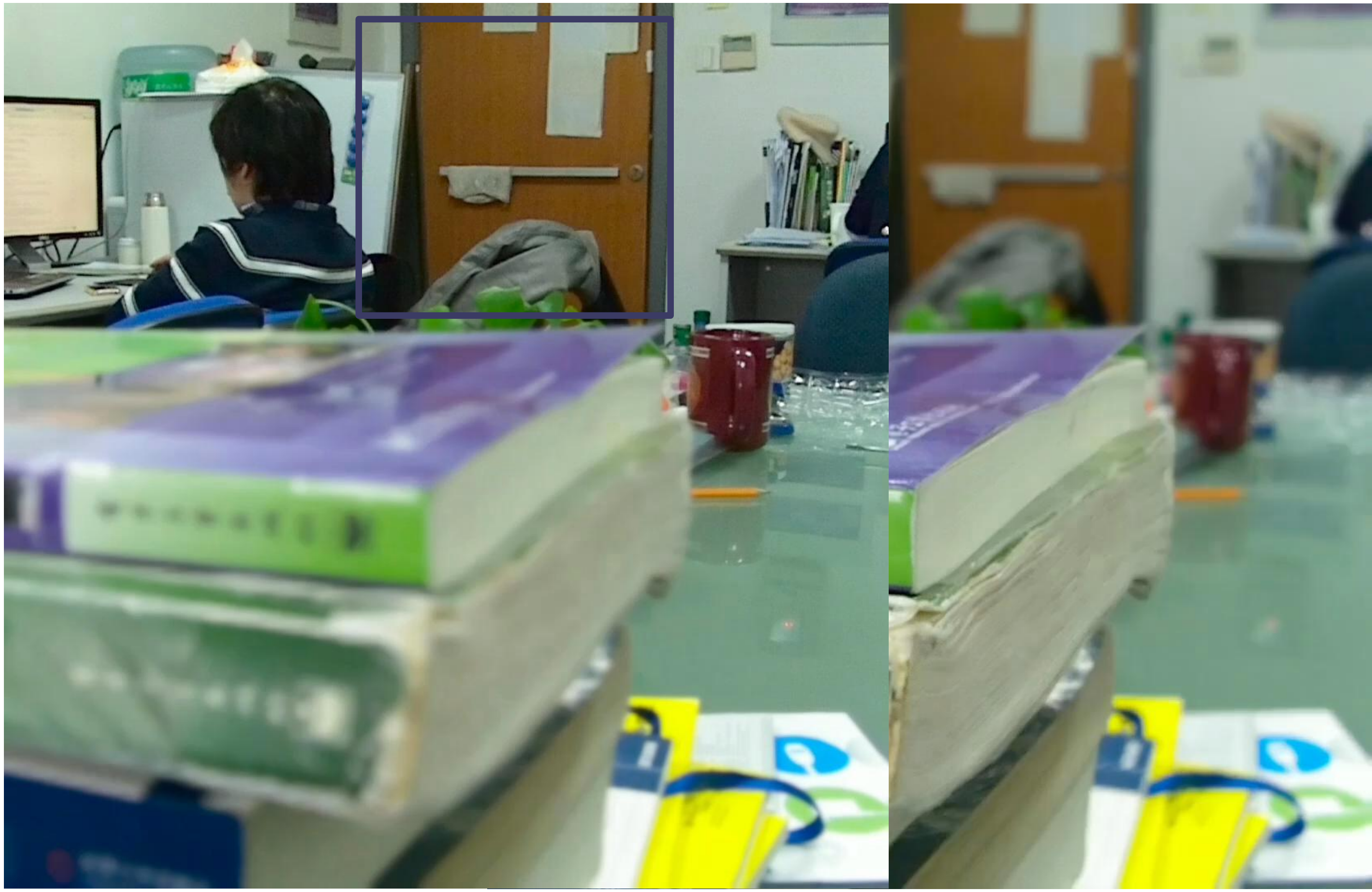
A New Kind of Camera-Lytro System

- The Lytro camera lets you create living pictures
- that you can endlessly refocus after you take them.
- See the light. All of it.
- Refocus pictures after you take them.
- Move the picture in any
  your perspective.

Light-Field Camera — Refocusing

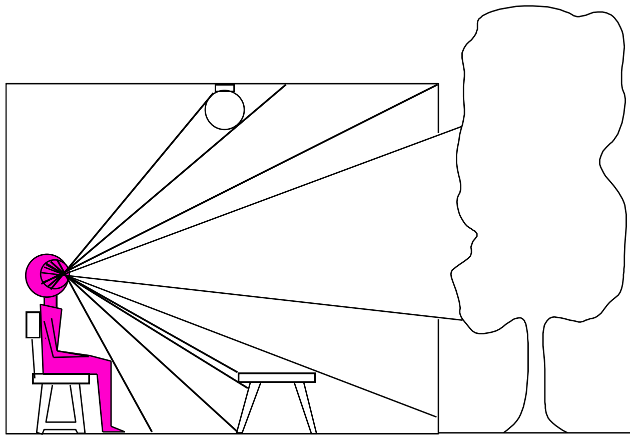
This video shows the refocusing results in different depths

Digital Refocusing on MIN



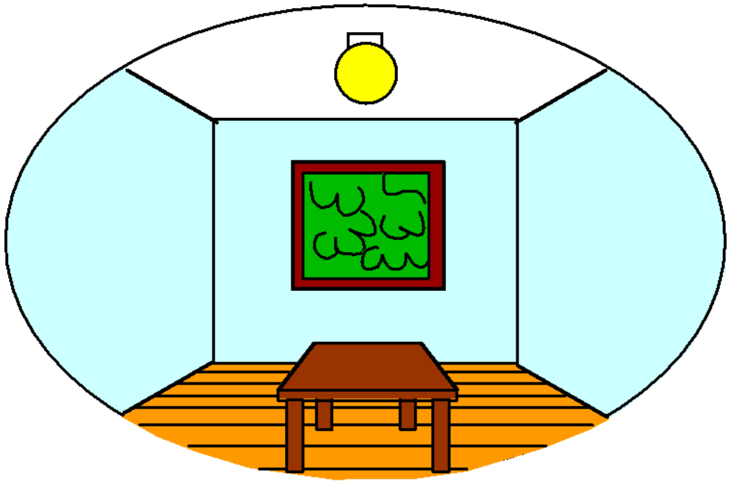
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image



Slide source: Seitz

Projection can be tricky…

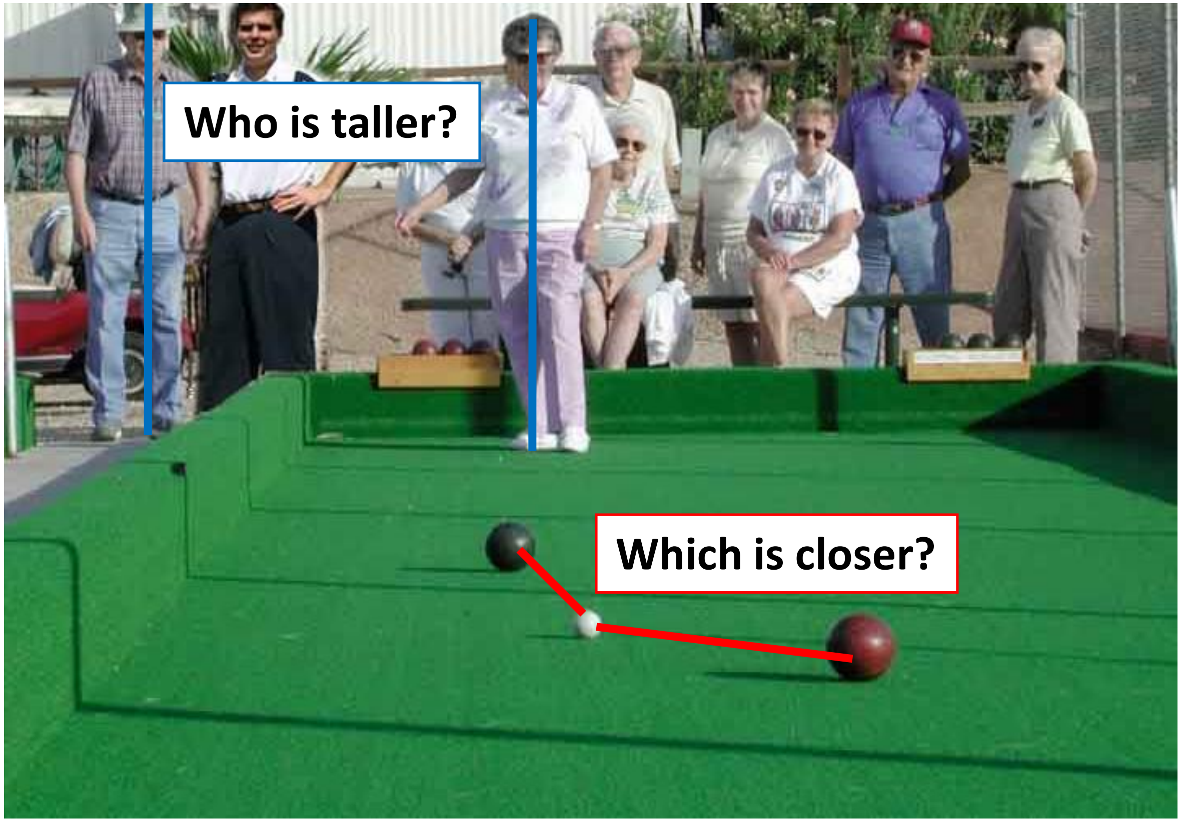
Slide source: Seitz

Projection can be tricky…

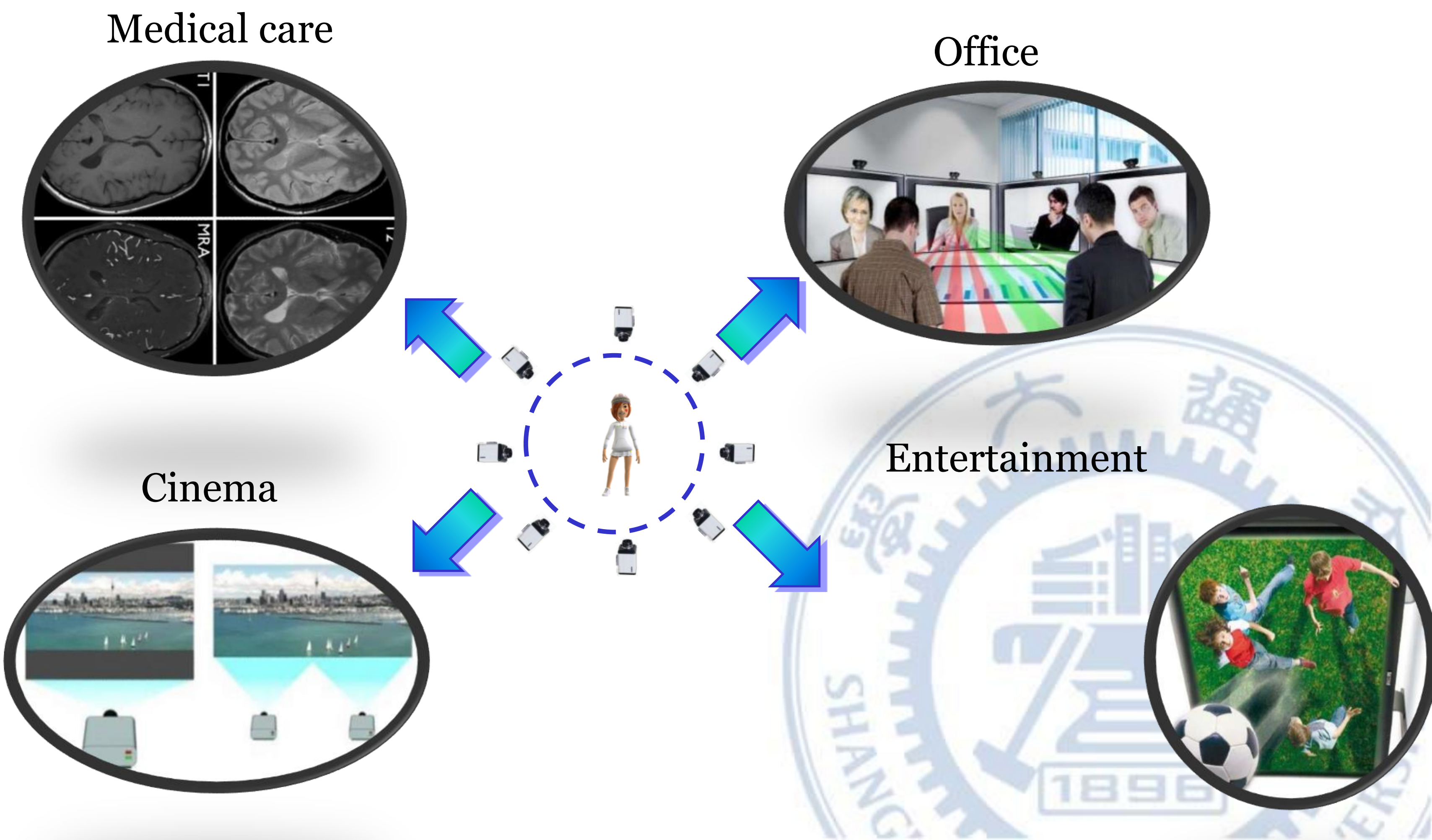
Projective Geometry

What is lost?

- Length

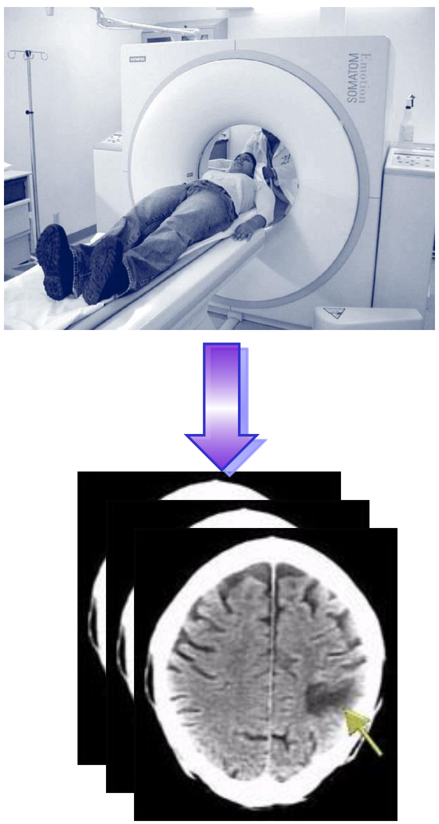


3D Applications

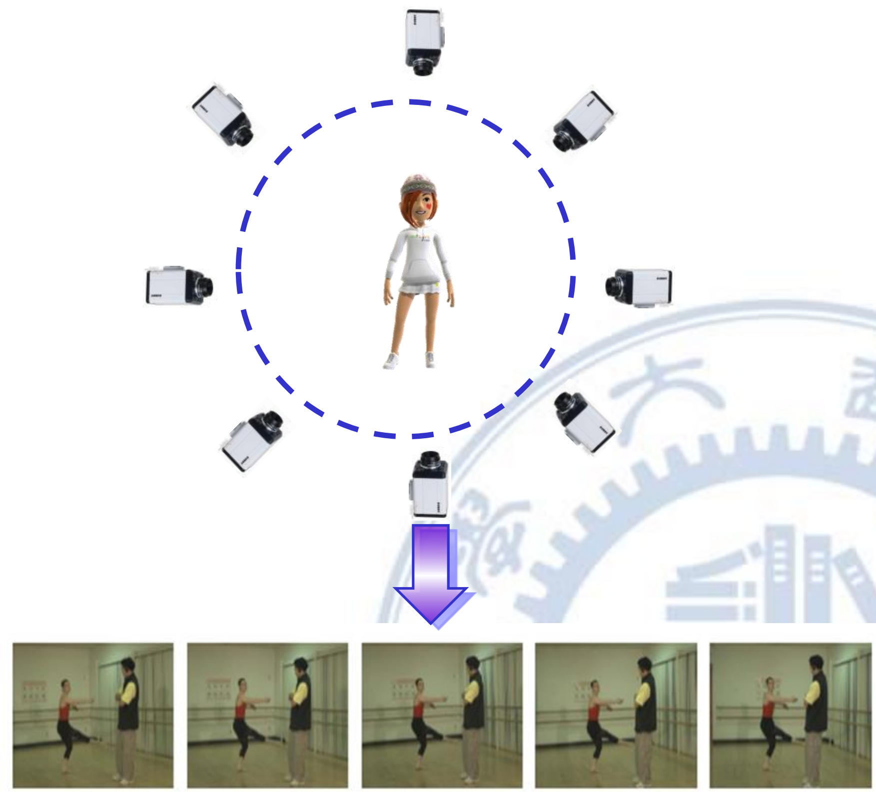


3D Data Capture

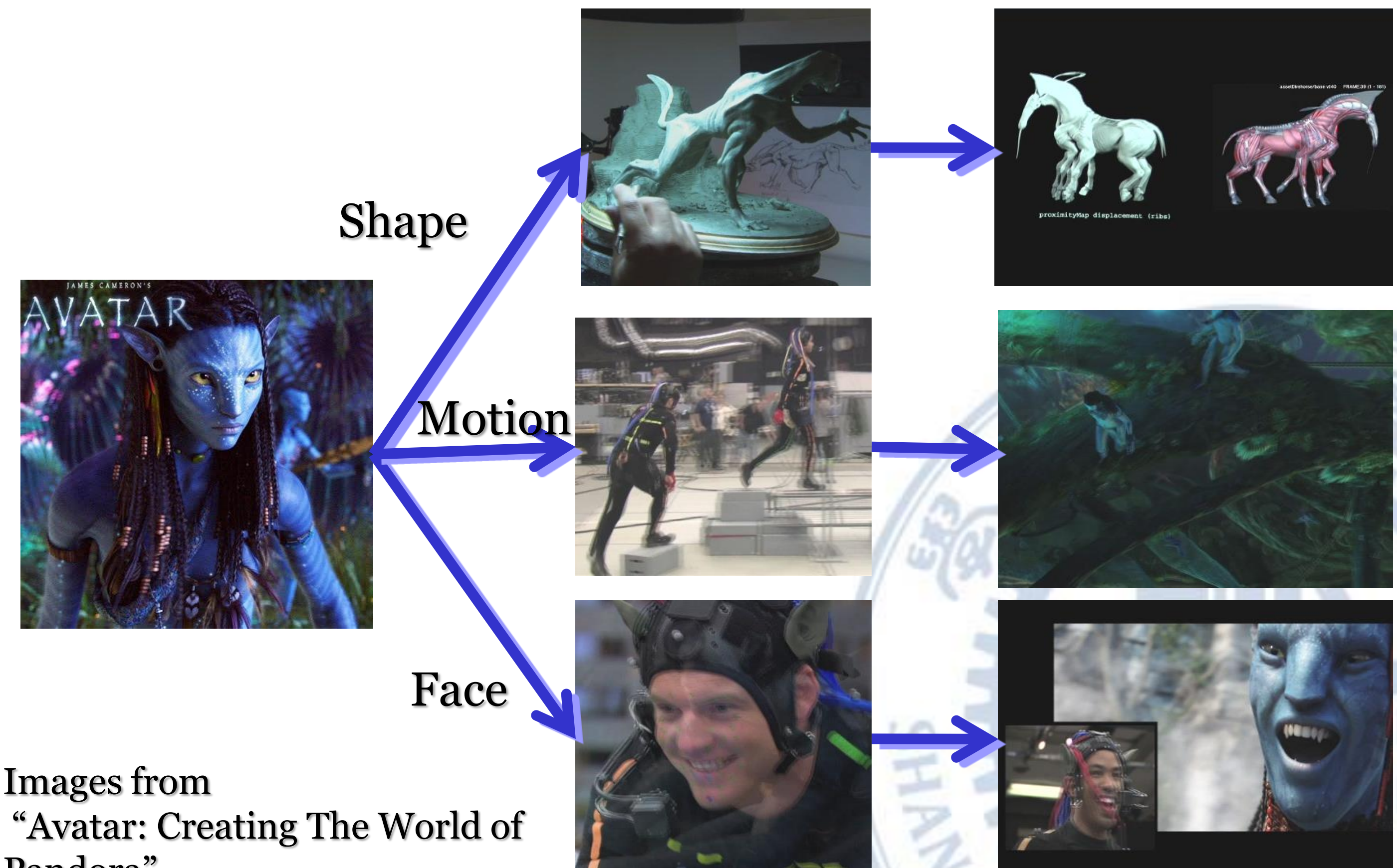
CT / MRI scanner



Multi-view

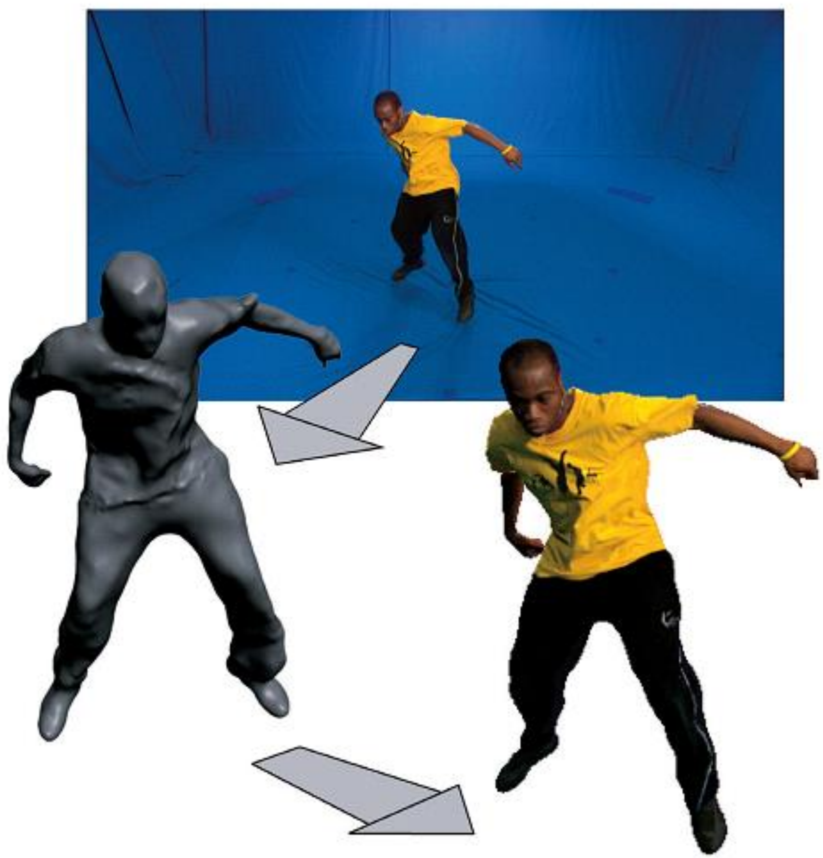


3D Capture Technique in Avatar



Images from
"Avatar: Creating The World of Pandora"

3D Surface Reconstruction



Surface reconstruction
Using Visual-Hull and geometric constraints

Automatic 3D reconstruction from internet photo collections

"Statue of Liberty" "Half Dome, Yosemite" "Colosseum, Rome"

Flickr photos

3D model

Seam Carving

Seam Carving

Seam Carving

Seam Carving

Seam Carving

Seam Carving

Seam Carving

Seam Carving

Simple object removal: the user marks a region for removal (green), and possibly a region to protect (red), on the original image (see inset in left image). On the right image, consecutive vertical seam were removed until no 'green' pixels were left.

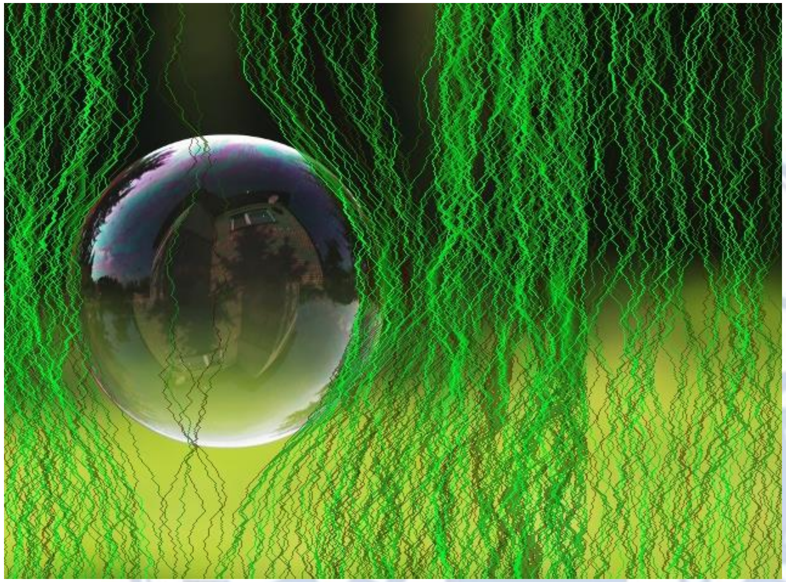
Seam Carving

Find the missing shoe!

Object removal: In this example, in addition to removing the object (one shoe), the image was enlarged back to its original size. Note that this example would be difficult to accomplish using in-painting or texture synthesis.

Liquid Rescale

- Calculate the weight/density/energy of each pixel
- Generate a list of seams



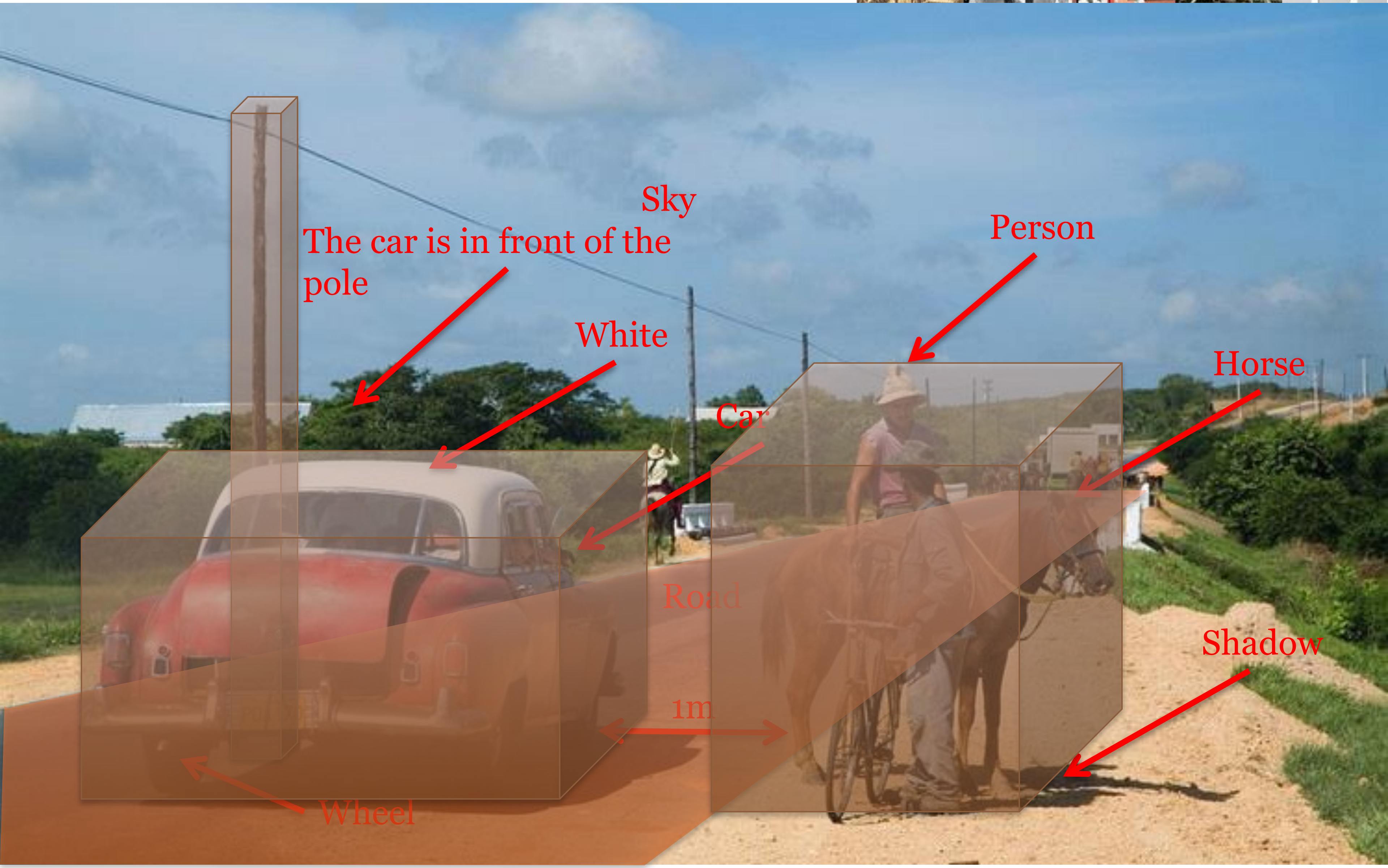
Liquid Rescale

- Calculate the weight/density/energy of each pixel
- Generate a list of seams

Why is vision difficult?

- What do computers see?

47	49	51	47	41	41	41	38	42	54	66	66	58	56	53	48	43	43	45	47	50	47	47	47
45	44	39	38	37	48	67	95	138	151	156	157	165	157	125	79	36	38	47	48	48	43	38	36
43	35	31	45	64	109	155	179	178	160	142	132	146	187	195	170	133	86	45	46	51	41	36	32
33	24	24	47	88	149	135	136	160	170	166	135	111	153	169	169	109	113	86	57	49	46	40	36
22	19	22	47	122	131	99	120	204	199	185	150	119	152	159	173	110	80	83	82	63	58	45	42
22	20	24	60	114	108	123	191	215	212	198	169	156	169	168	172	151	115	91	77	82	59	53	53
20	19	29	86	127	87	169	223	219	218	212	182	178	190	194	185	169	108	88	85	74	55	52	51
20	20	26	131	138	129	214	228	224	222	221	206	207	208	203	193	177	136	88	87	72	54	44	42
24	23	28	130	125	152	226	224	222	223	217	218	214	201	185	168	164	114	70	39	45	47	39	34
29	26	25	104	92	123	220	226	230	228	218	213	210	193	152	118	136	97	50	26	39	41	36	33
26	24	25	66	95	140	222	223	228	225	218	208	205	181	140	97	101	121	71	35	78	51	40	37
26	30	24	51	149	179	224	221	218	215	205	204	210	191	140	108	107	127	112	43	46	42	39	40
27	34	30	23	142	198	210	226	233	220	205	204	222	210	175	154	134	125	137	51	54	55	44	34
26	32	29	18	124	197	178	174	140	113	182	183	174	112	98	74	34	69	126	54	53	78	59	41
30	27	26	19	114	197	207	138	73	43	167	191	49	29	139	66	33	76	92	60	85	50	42	40
26	25	23	18	91	198	220	221	184	133	210	214	40	112	210	129	120	105	81	62	60	28	22	30
23	19	16	13	53	201	211	227	220	227	226	216	75	72	196	190	130	58	62	58	32	21	24	26
18	14	12	11	13	93	198	220	226	209	219	218	121	34	148	170	53	37	50	25	17	17	23	24
17	15	14	13	15	25	177	203	189	151	223	219	139	59	33	78	30	39	45	26	22	21	16	38
12	14	17	13	15	11	125	201	149	194	223	203	67	19	15	22	33	43	55	37	29	28	31	68
10	13	14	11	16	15	58	196	170	193	213	175	123	34	19	48	37	93	35	32	30	38	93	118
17	19	19	20	31	35	30	145	191	201	215	182	134	47	66	89	45	196	45	16	52	98	141	149
25	28	34	34	28	32	20	105	216	215	213	187	168	130	73	26	148	195	34	12	21	76	121	123
31	36	30	26	29	42	20	77	220	215	221	213	185	131	37	117	201	85	56	11	16	10	22	38
24	20	21	40	43	42	24	106	190	235	212	188	134	85	138	178	45	89	40	13	19	13	19	21


Sky
The car is in front of the pole
White
Person
Horse
Car
Road
Shadow
1m
Wheel

Visual Cues

- People use information from various visual cues for recognition (e.g., color, shape, texture etc.)
  - How important is each visual cue?
  - How do we combine information from various visual cues?

Color Cues

Texture Cues

Shape Cues

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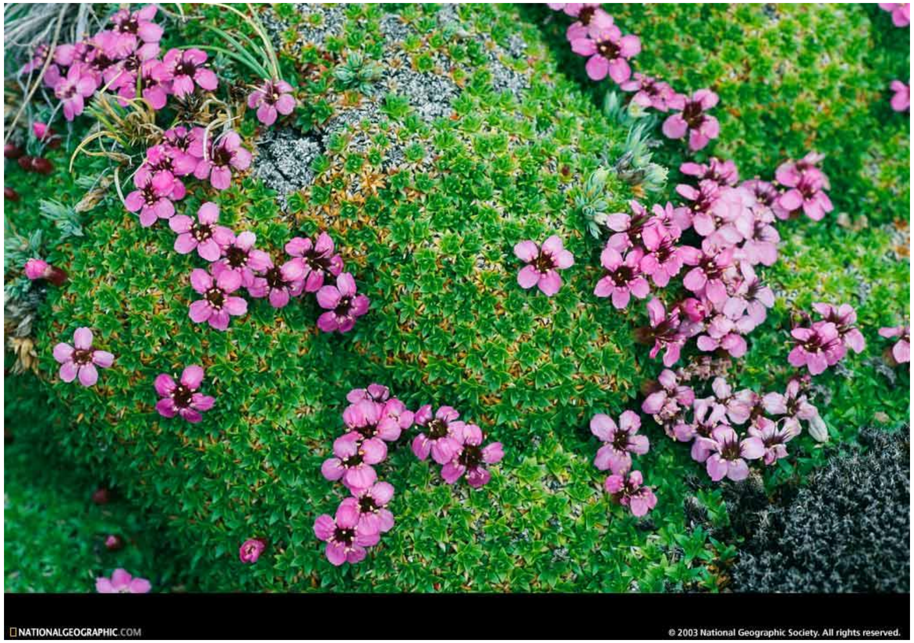
Grouping Cues

Similarity (color, texture, proximity)



Depth Cues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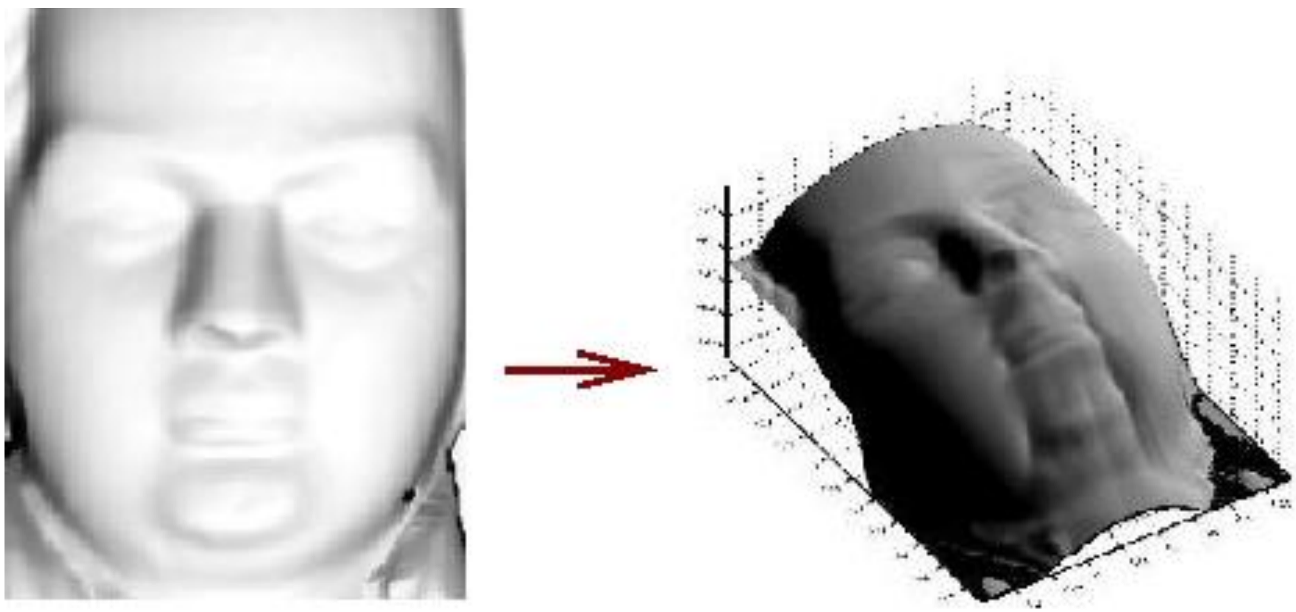
Shading Cues



a) Image

b) 3D surface reconstructed from the single image a)

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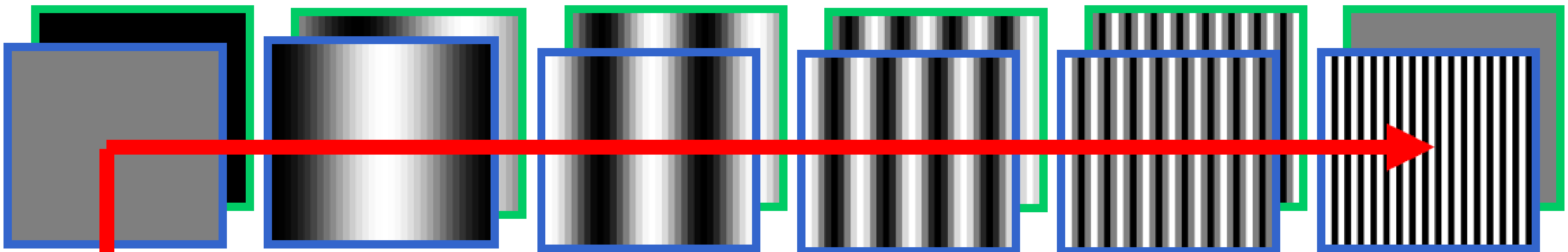
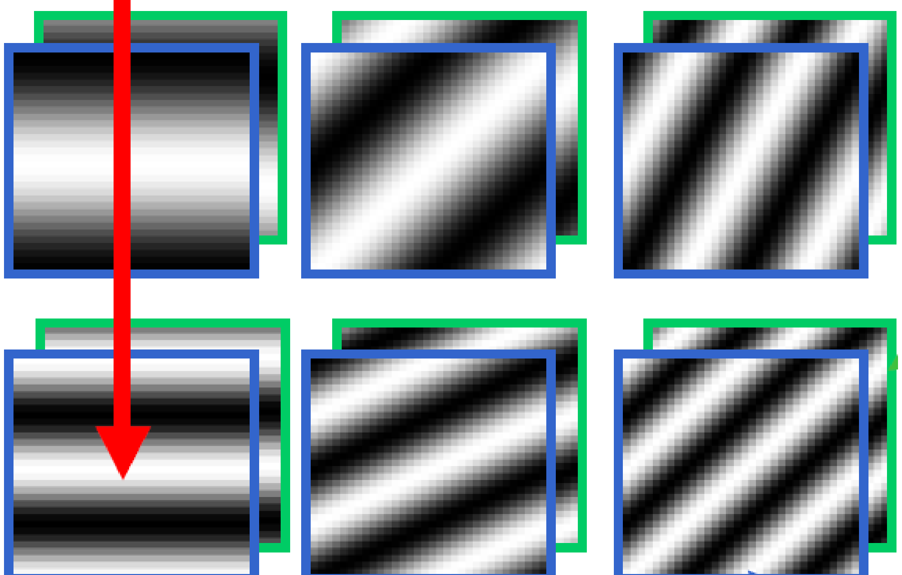
Frequency Cues

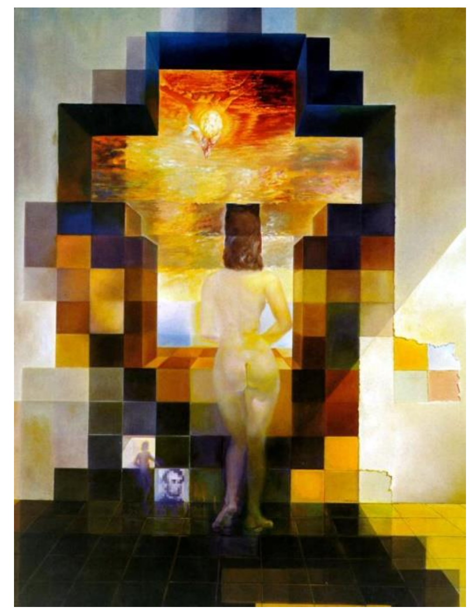
Salvador Dali
"Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln", 1976

A nice set of basis

Teases away fast vs. slow changes in the image.







This change of basis has a special name…

Frequency cues in Fourier Transform

Example:
cheetah pic

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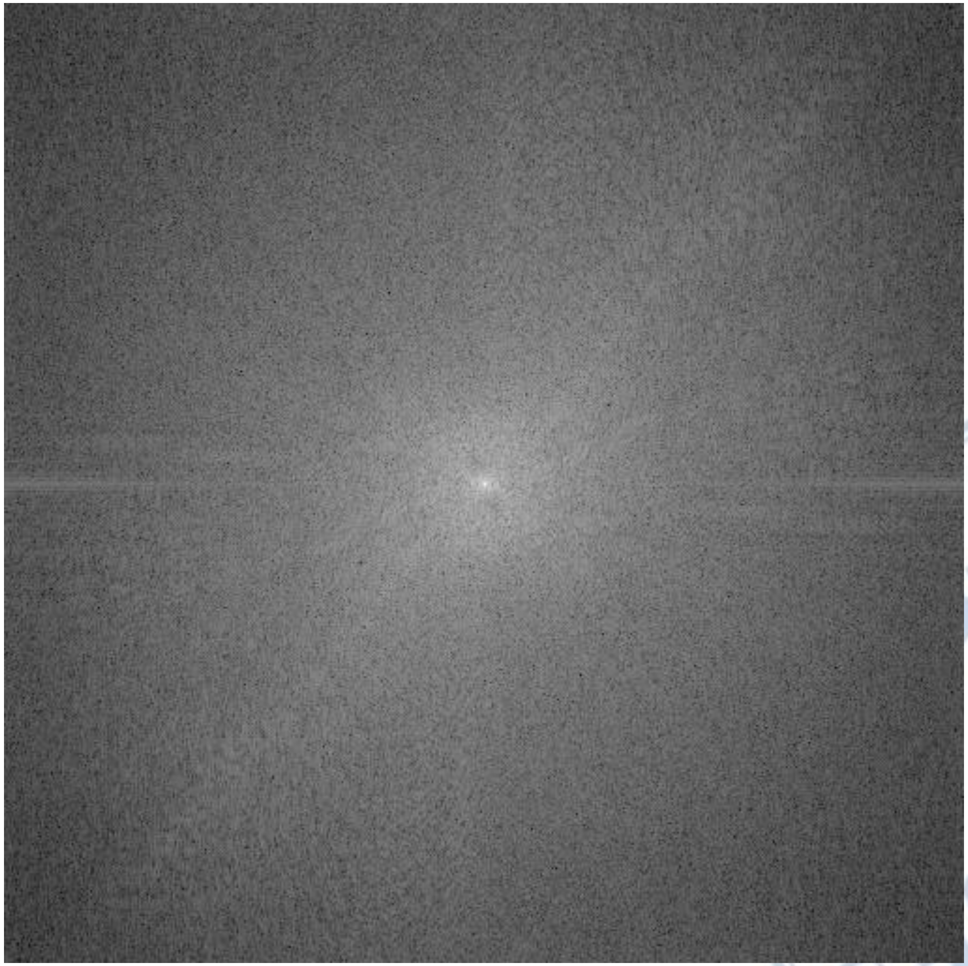
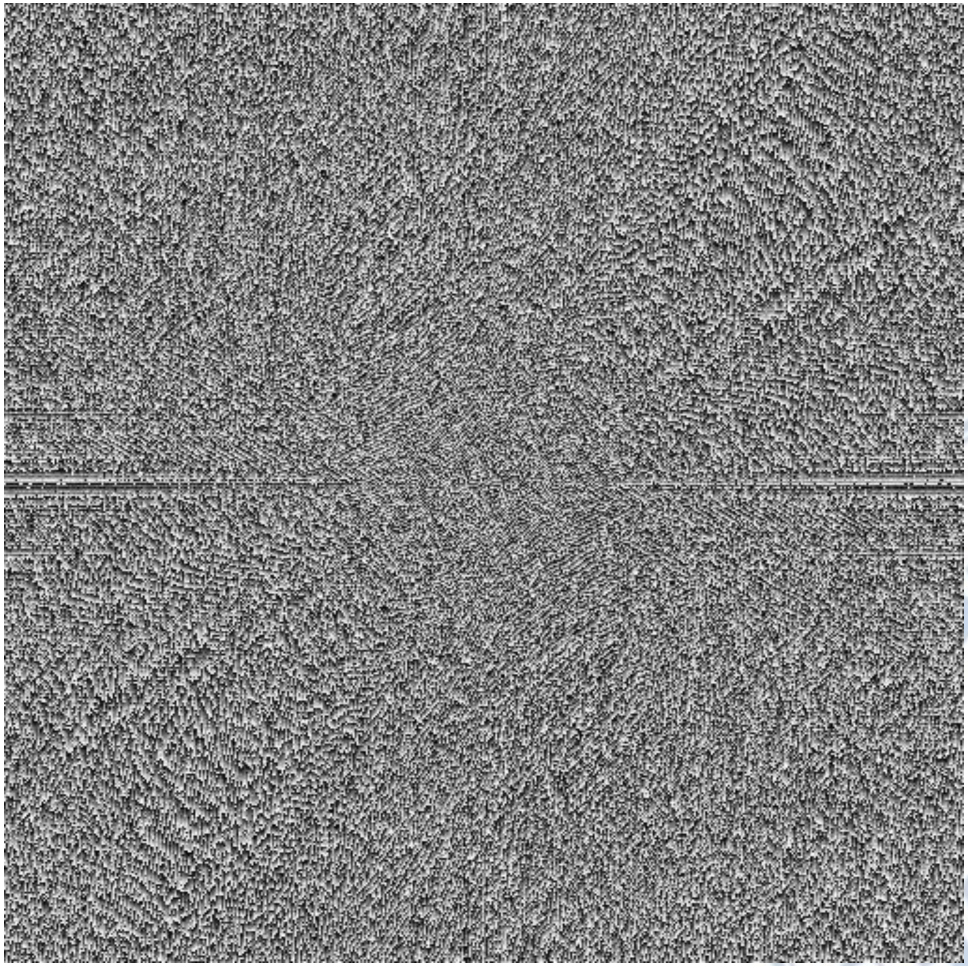
Frequency cues in Fourier Transform

This is the magnitude transform of the cheetah pic



Frequency cues in Fourier Transform

This is the phase transform of the cheetah pic

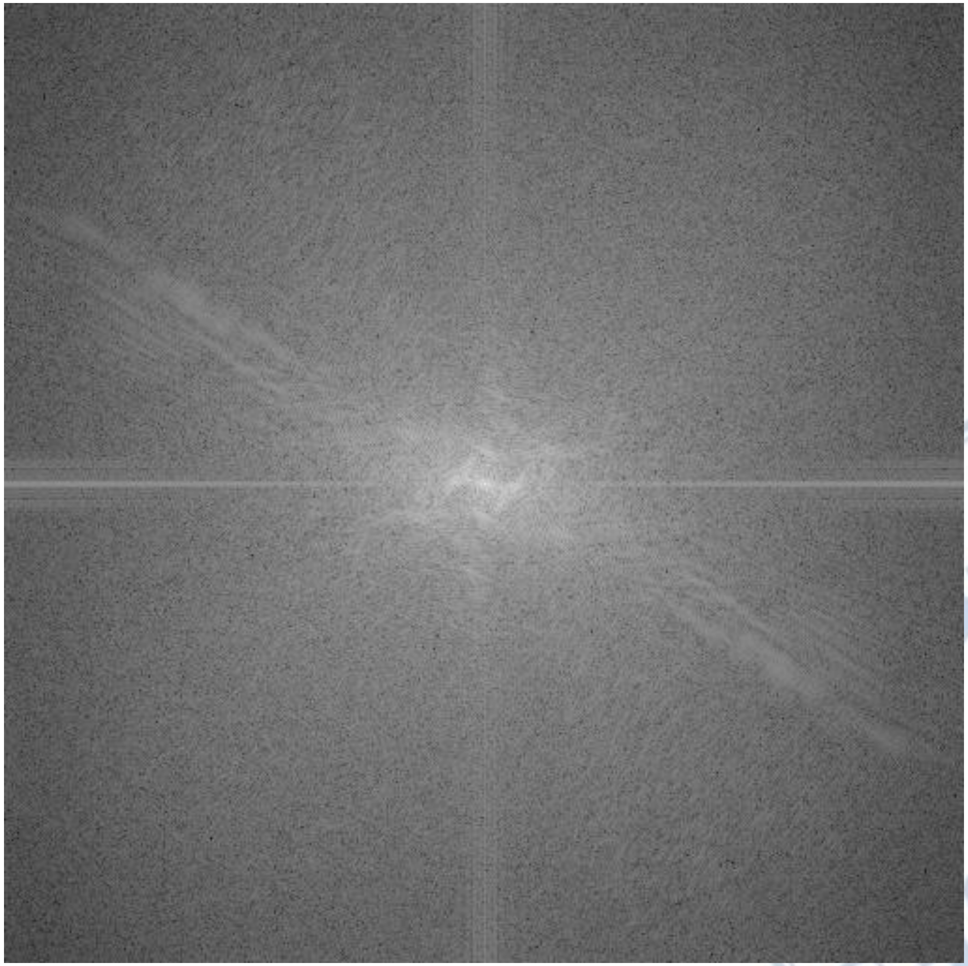


Frequency cues in Fourier Transform

Example:
zebra pic

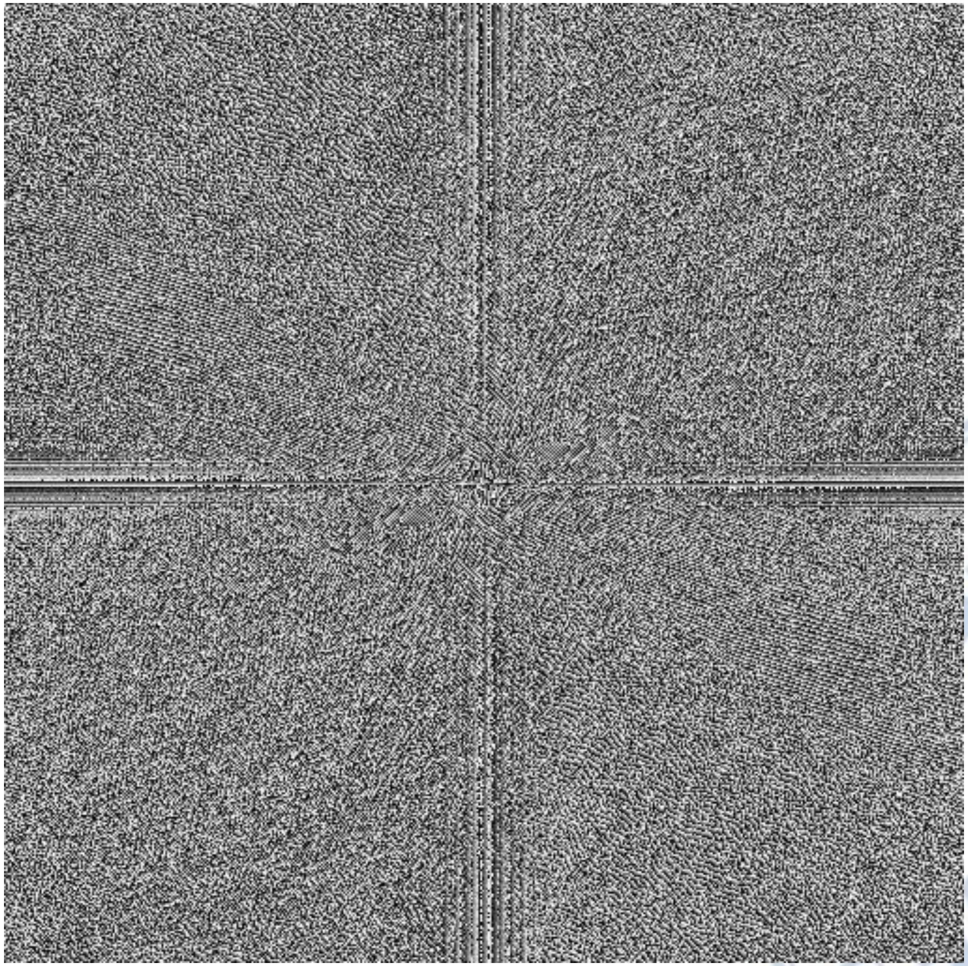
Frequency cues in Fourier Transform

This is the magnitude transform of the zebra pic



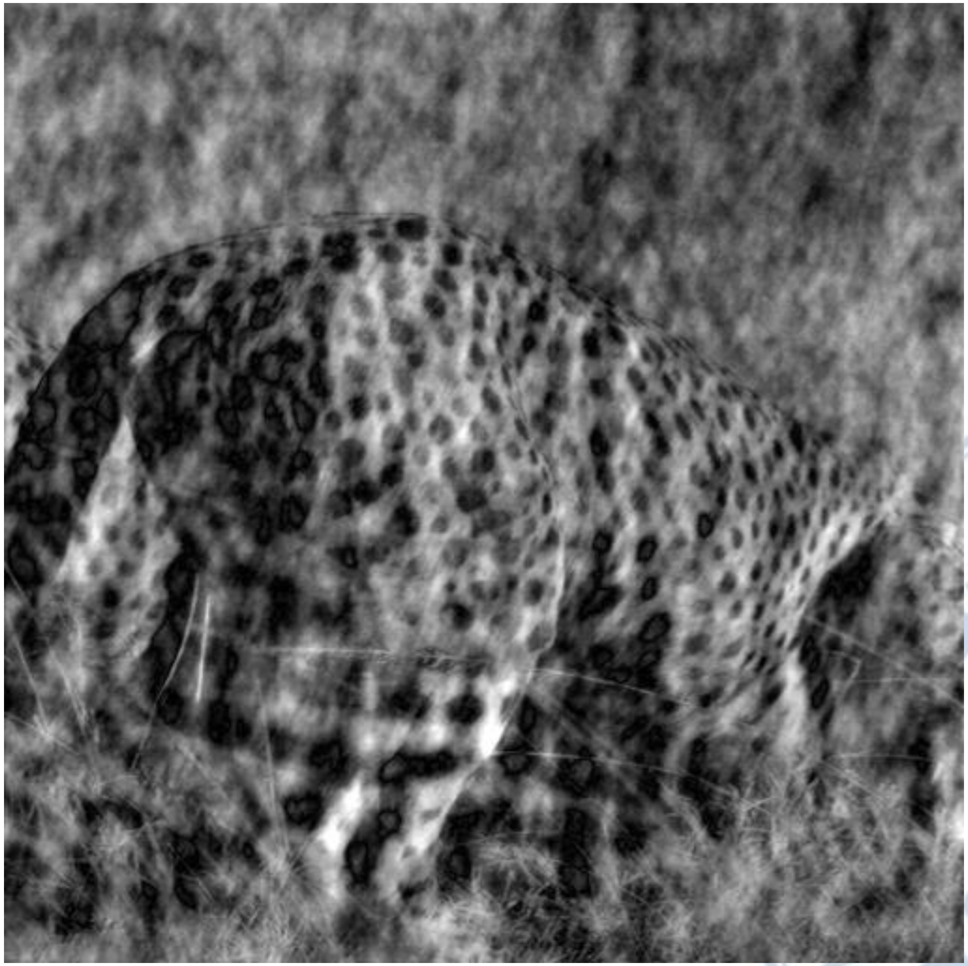
Frequency cues in Fourier Transform

This is the phase transform of the zebra pic



Frequency cues in Fourier Transform

Reconstruction with cheetah phase, zebra magnitude



Frequency cues in Fourier Transform

Reconstruction with zebra phase, cheetah magnitude

Phase carries location information

Practice with linear filters

Original

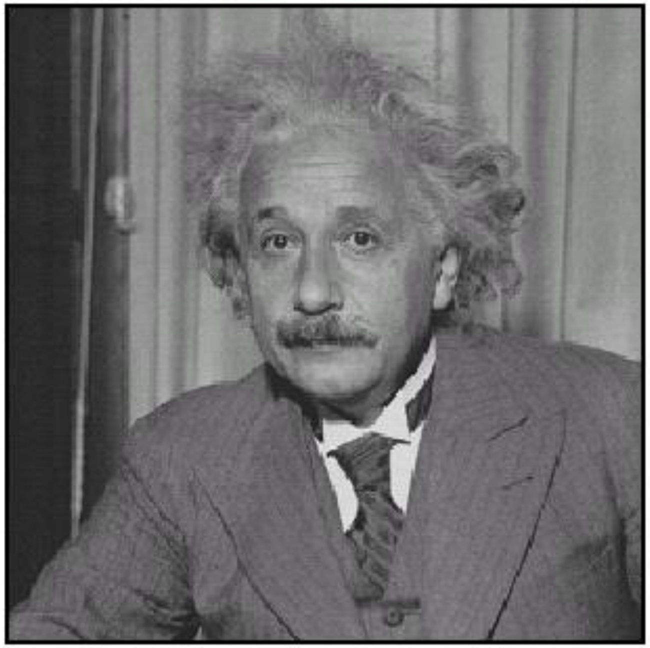
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpening filter

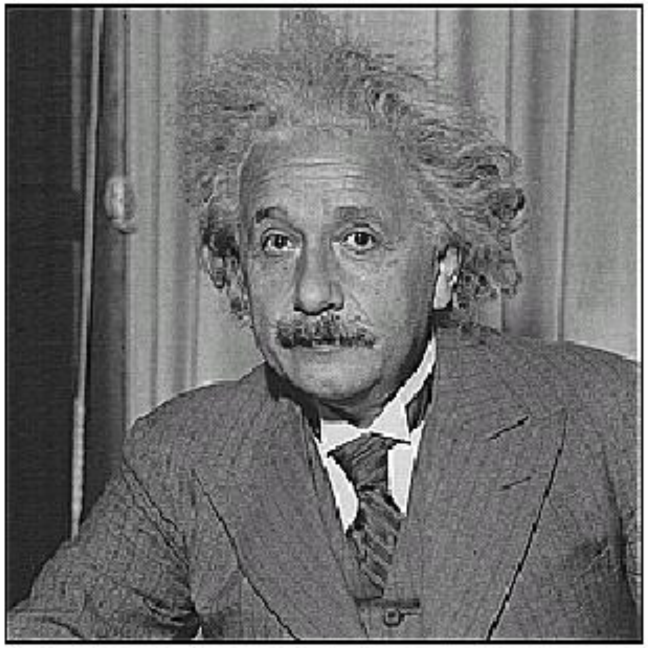
- Accentuates differences with local average

Source: D. Lowe

Sharpening



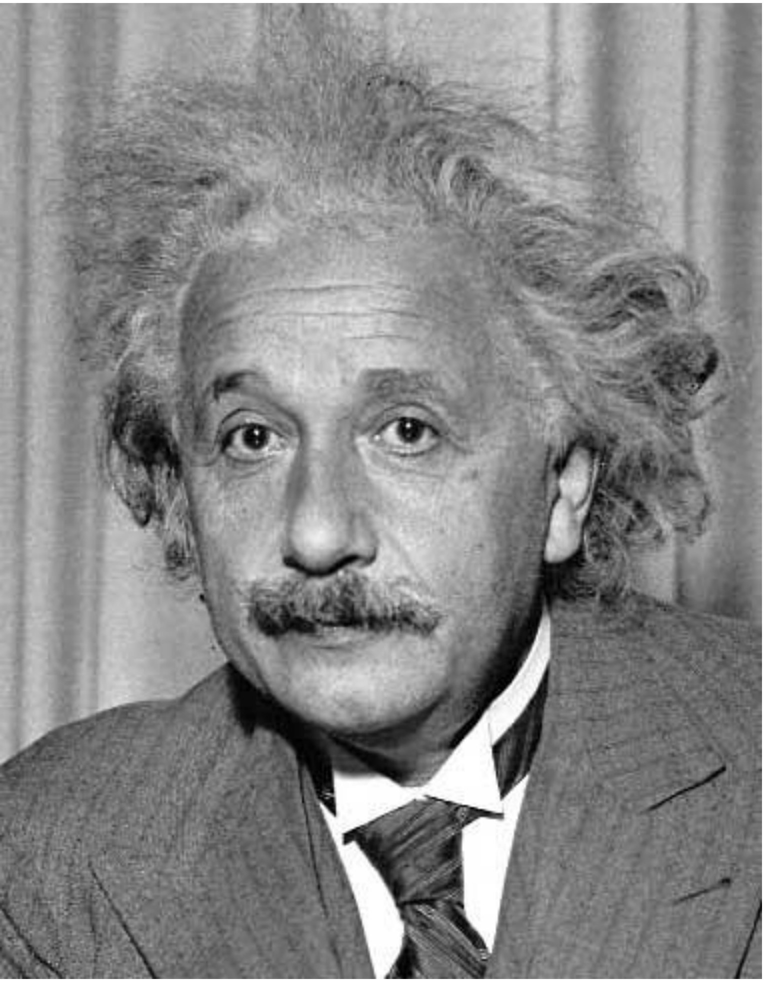
before



after

Source: D. Lowe

Other filters

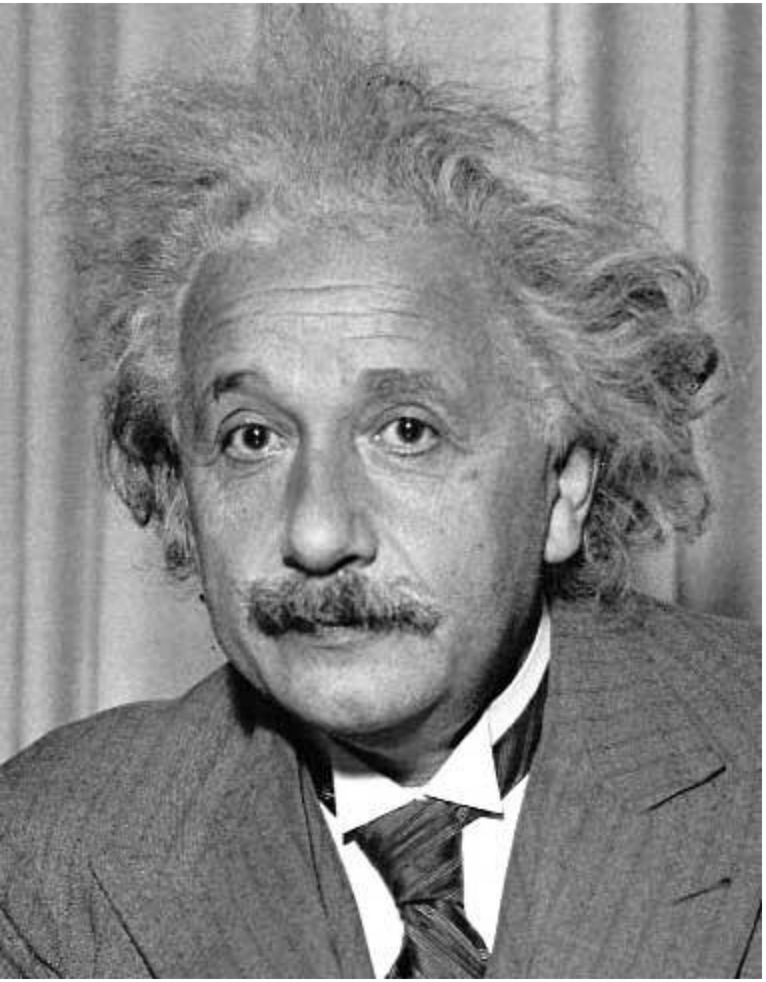


1	0	-1
2	0	-2
1	0	-1

Sobel

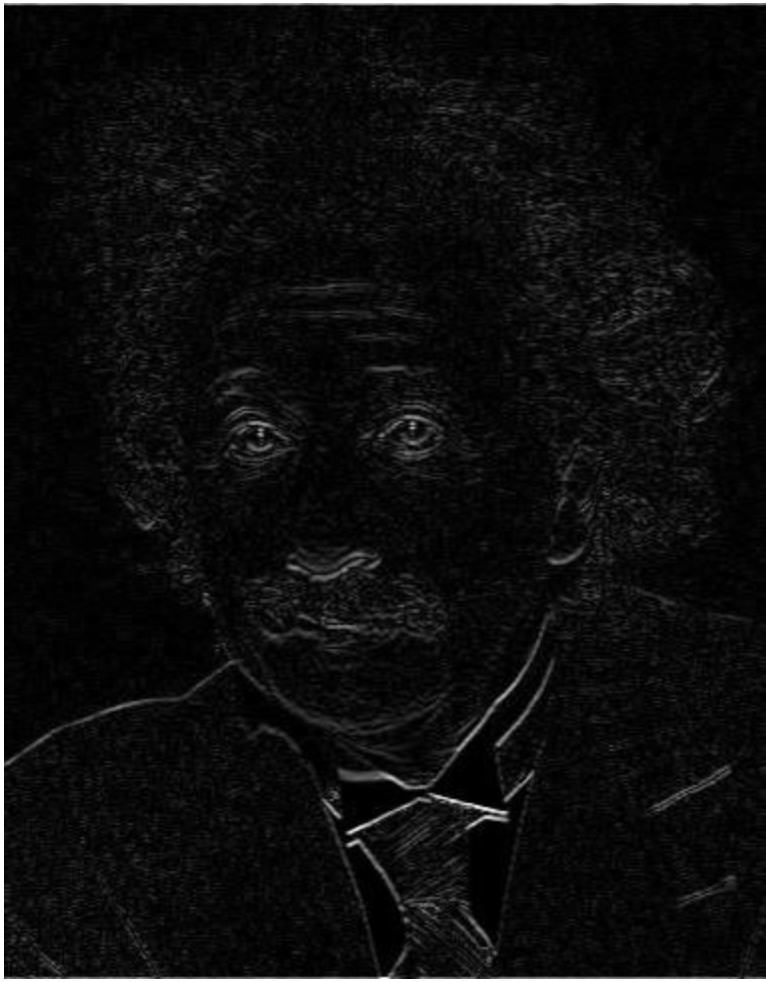
Vertical Edge
(absolute value)

Other filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

Filtering vs. Convolution

- 2d filtering
  - `h=filter2(g,f);` or `h=imfilter(f,g);` (g=filter, f=image)

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

- 2d convolution
  - `h=conv2(g,f);`

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m-k, n-l]$$

Key properties of linear filters

Linearity:

$\texttt{filter}(f_1 + f_2) = \texttt{filter}(f_1) + \texttt{filter}(f_2)$

Shift invariance: same behavior regardless of pixel location

`filter(shift(f)) = shift(filter(f))`

Any linear, shift-invariant operator can be represented as a convolution

Source: S. Lazebnik

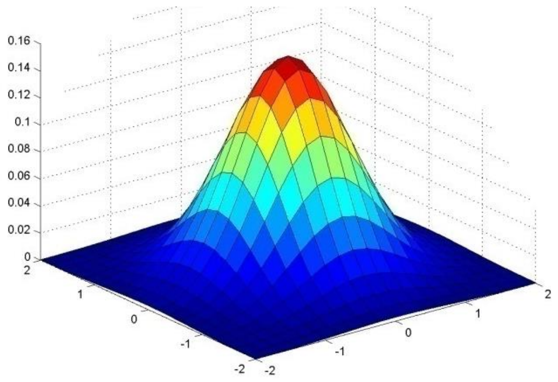
More properties

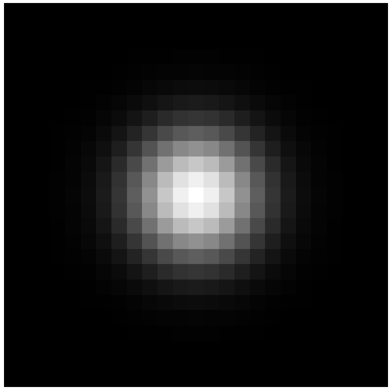
- Commutative: $a * b = b * a$
  - Conceptually no difference between filter and signal
- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: a * $(b_1 * b_2 * b_3)$
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k (a * b)$
- Identity: unit impulse e = [0, 0, 1, 0, 0],
  $a * e = a$

Source: S. Lazebnik

Important filter: Gaussian

- Weight contributions of neighboring pixels by nearness





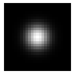
0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

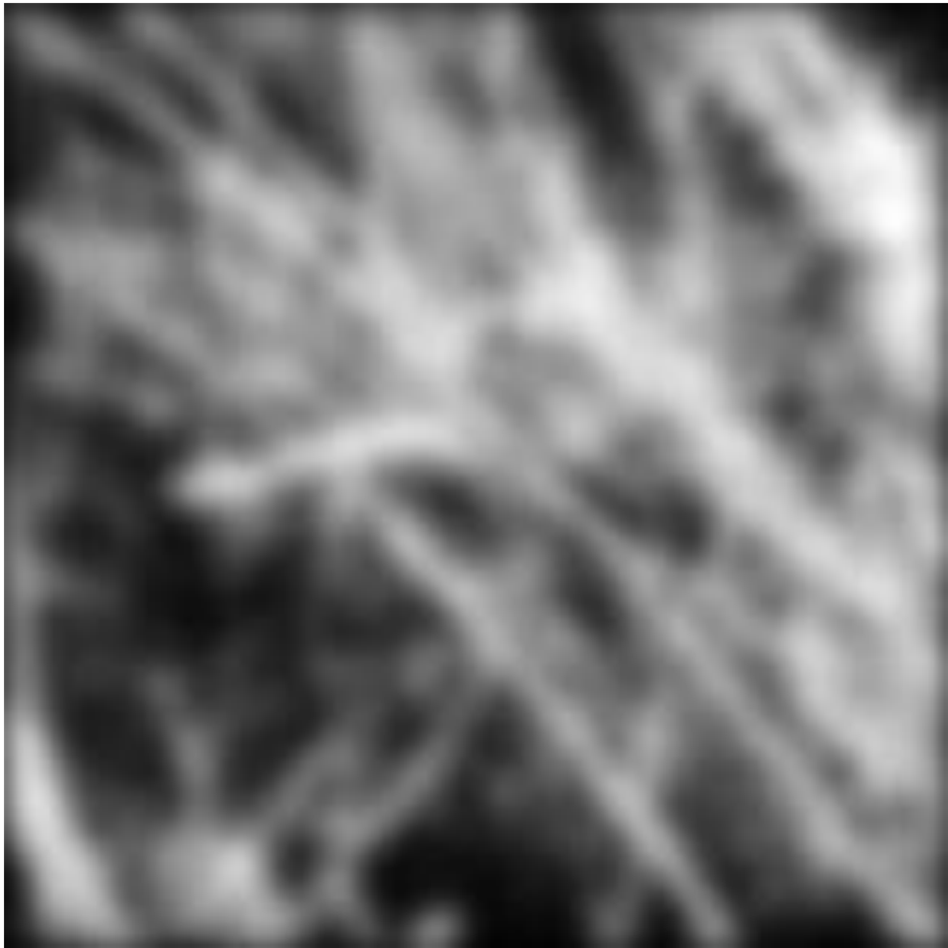
5 x 5, σ = 1

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

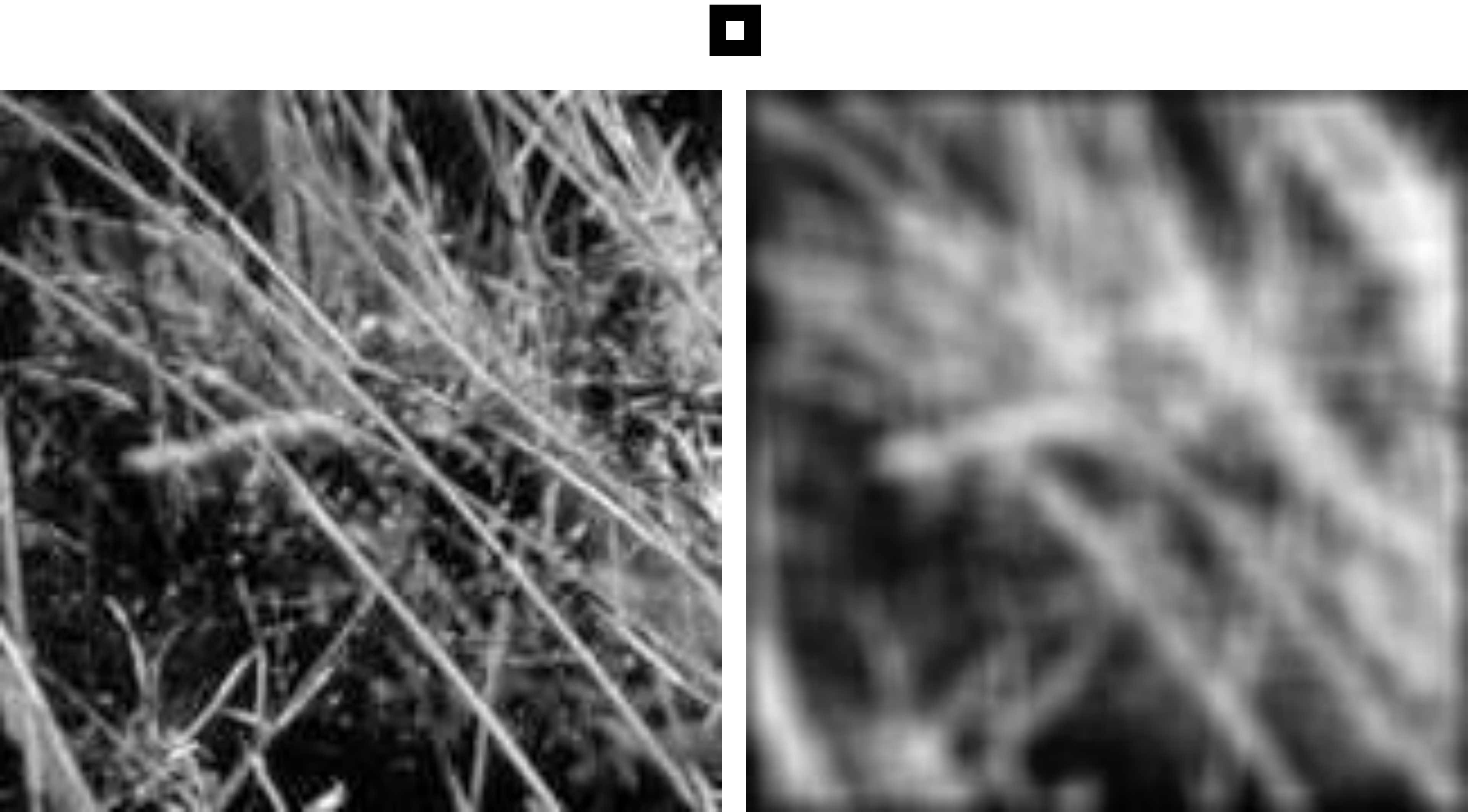
Slide credit: Christopher Rasmussen

Smoothing with Gaussian filter





Smoothing with box filter



Gaussian filters

- Remove “high-frequency” components from the image (low-pass filter)
  – Images become more smooth
- Convolution with self is another Gaussian
  – So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  – Convolving two times with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
  – Factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}}\right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Separability example

2D convolution
(center location only)

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix}$$

The filter factors
into a product of 1D
filters:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Perform convolution
along rows:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix} = \begin{bmatrix} & 11 & \\ & 18 & \\ & 18 & \end{bmatrix}$$

Followed by convolution
along the remaining column:

Source: K. Grauman

Separability

- Why is separability useful in practice?

Some practical matters

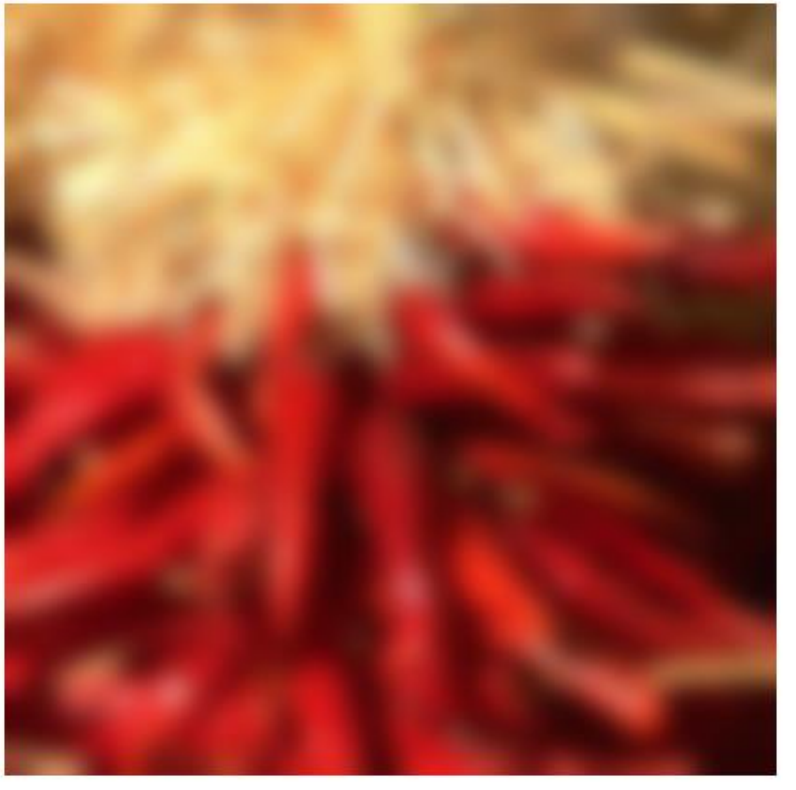
Practical matters

How big should the filter be?

- Values at edges should be near zero
- Rule of thumb for Gaussian: set filter half-width to about 3 σ

Practical matters

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge



Source: S. Marschner

Q?

Practical matters

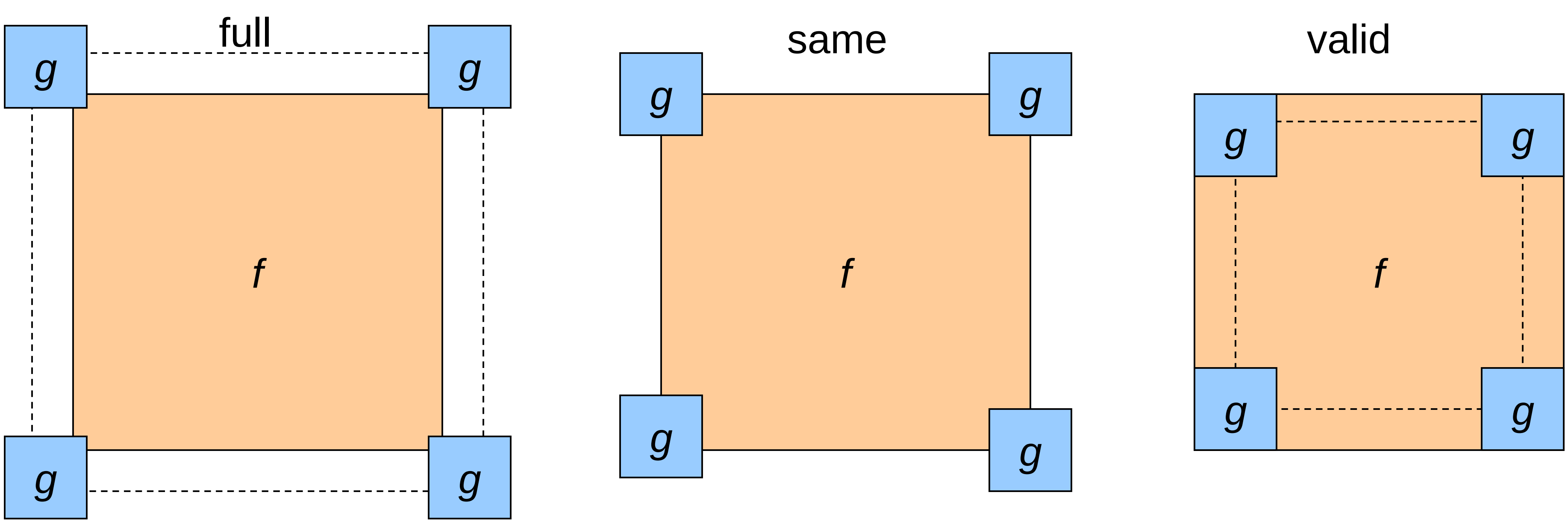
– methods (MATLAB):

- clip filter (black): imfilter(f, g, 0)
- wrap around: imfilter(f, g, ‘circular’)
- copy edge: imfilter(f, g, ‘replicate’)
- reflect across edge: imfilter(f, g, ‘symmetric’)

Source: S. Marschner

Practical matters

- What is the size of the output?
- MATLAB: filter2(g, f, *shape*)
  - *shape* = 'full': output size is sum of sizes of f and g
  - *shape* = 'same': output size is same as f
  - *shape* = 'valid': output size is difference of sizes of f and g

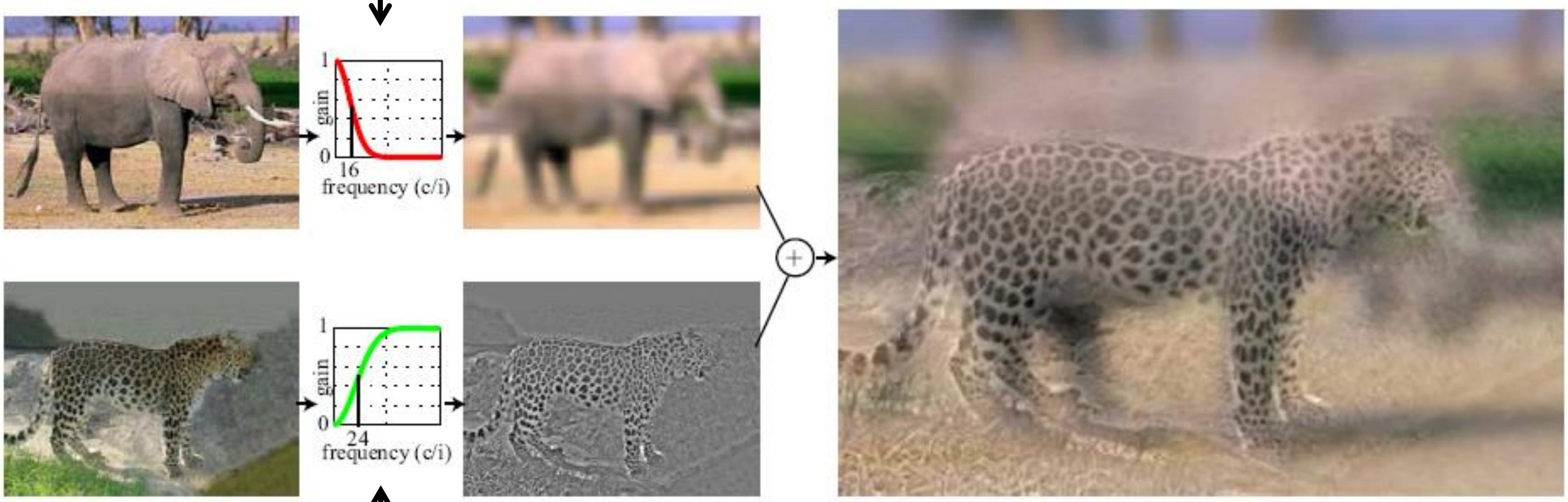


Source: S. Lazebnik

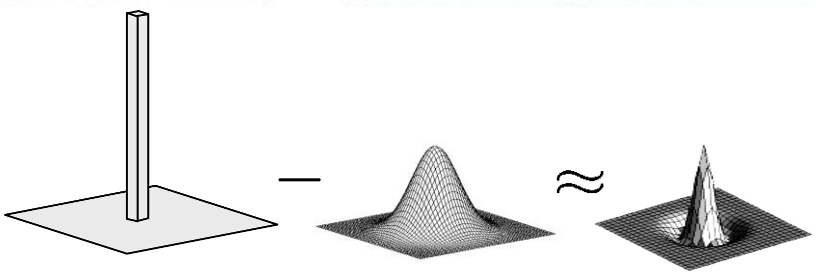
Project 1: Hybrid Images

A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006

Gaussian Filter!

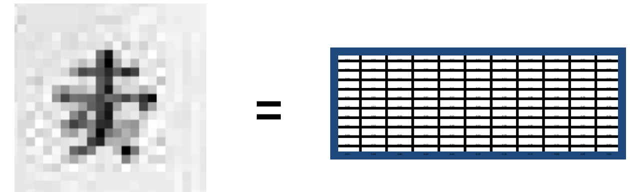


Laplacian Filter!



Take-home messages

- Image is a matrix of numbers



- Linear filtering is sum of dot product at each position
  - Can smooth, sharpen, translate (among many other uses)

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

- Be aware of details for filter size, extrapolation, cropping

Practice questions

1. Write down a 3x3 filter that returns a positive value if the average value of the 4-adjacent neighbors is less than the center and a negative value otherwise

2. Write down a filter that will compute the gradient in the x-direction:

```
gradx(y,x) = im(y,x+1)-im(y,x) for each x, y
```

Practice questions

Filtering Operator

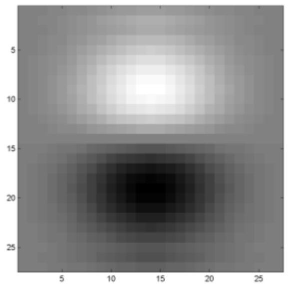
3. Fill in the blanks:

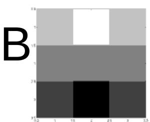
a) _ = D * B

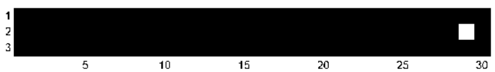
b) A = _ * _

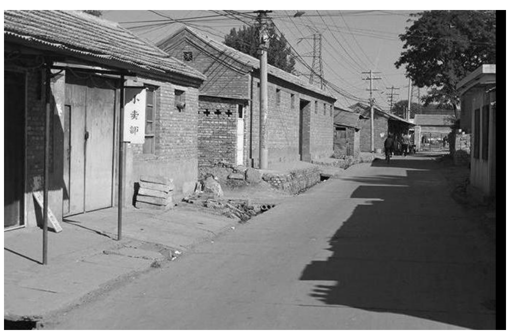
c) F = D * _

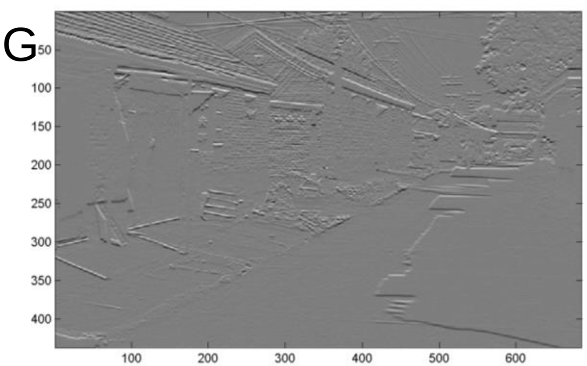
d) _ = D * D

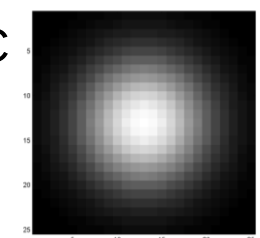
A


B


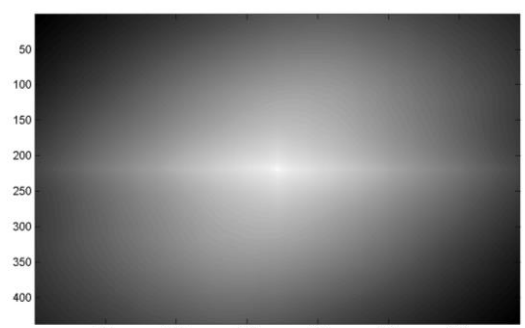
E


F


G


C


H


I


D


Learning representations/features

The traditional model of pattern recognition (since the late 50's)

- Fixed/engineered features (or fixed kernel) + trainable classifier

hand-crafted Feature Extractor → "Simple" Trainable Classifier →

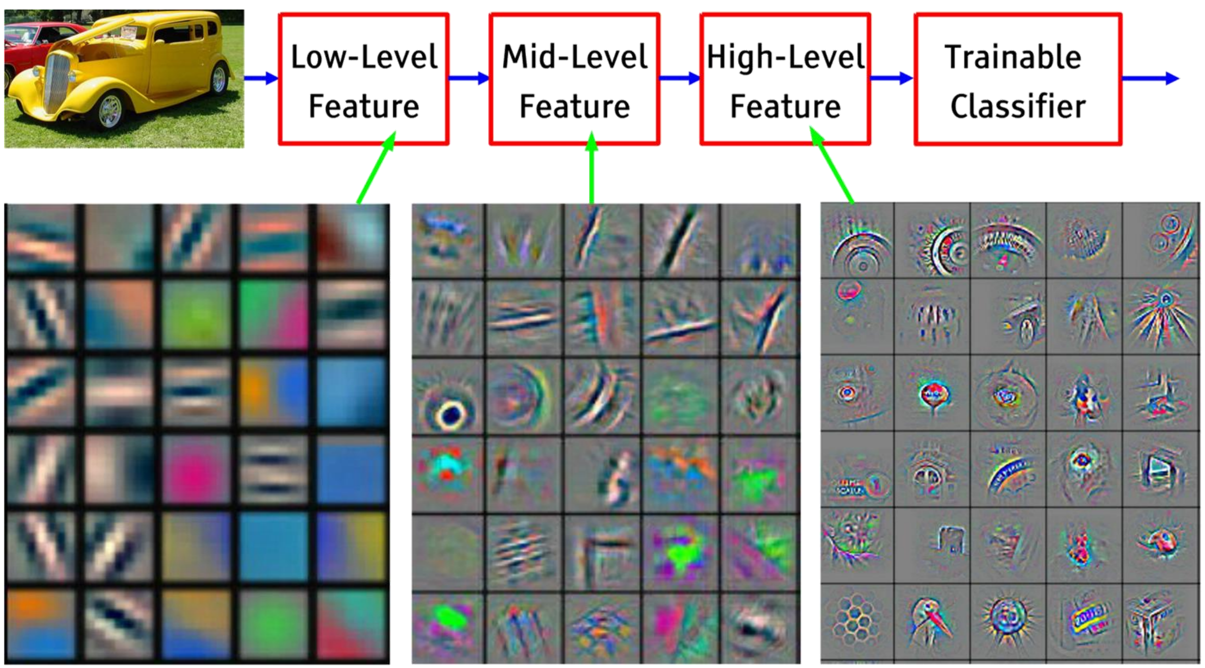
End-to-end learning / Feature learning / Deep learning

- Trainable features (or kernel) + trainable classifier

Trainable Feature Extractor → Trainable Classifier →

Deep Learning: Learning hierarchical representations

It's deep if it has more than one stage of non-linear feature transformation.



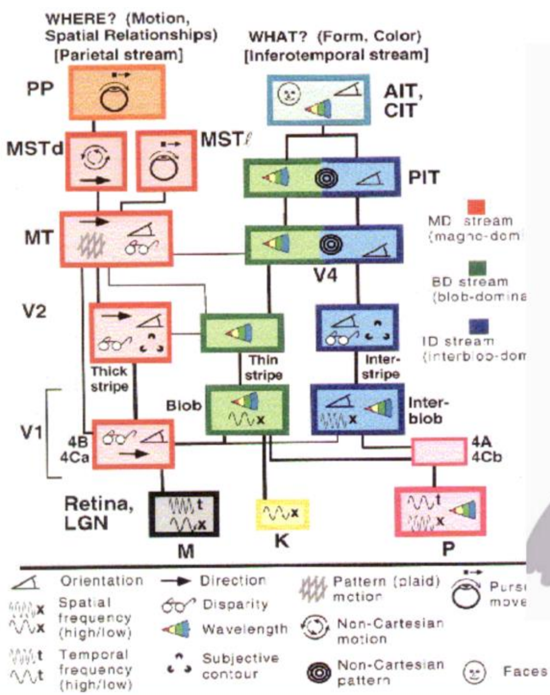
Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

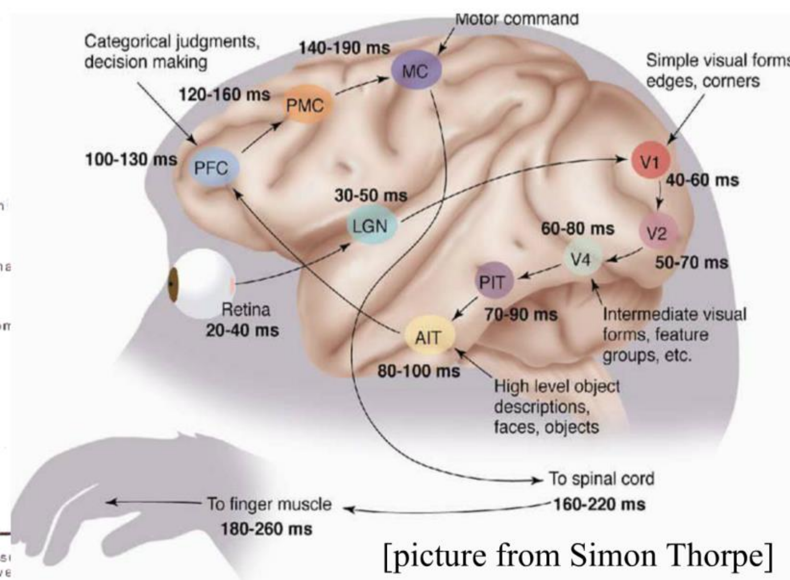
Why Deep Learning?

- How does the cortex learn perception?

The Mammalian Visual Cortex is Hierarchical

- The ventral (recognition) pathway in the visual cortex has multiple stages
- Retina-LGN- V1 - V2 - V4 - PIT - AIT ....
- Lots of intermediate representations

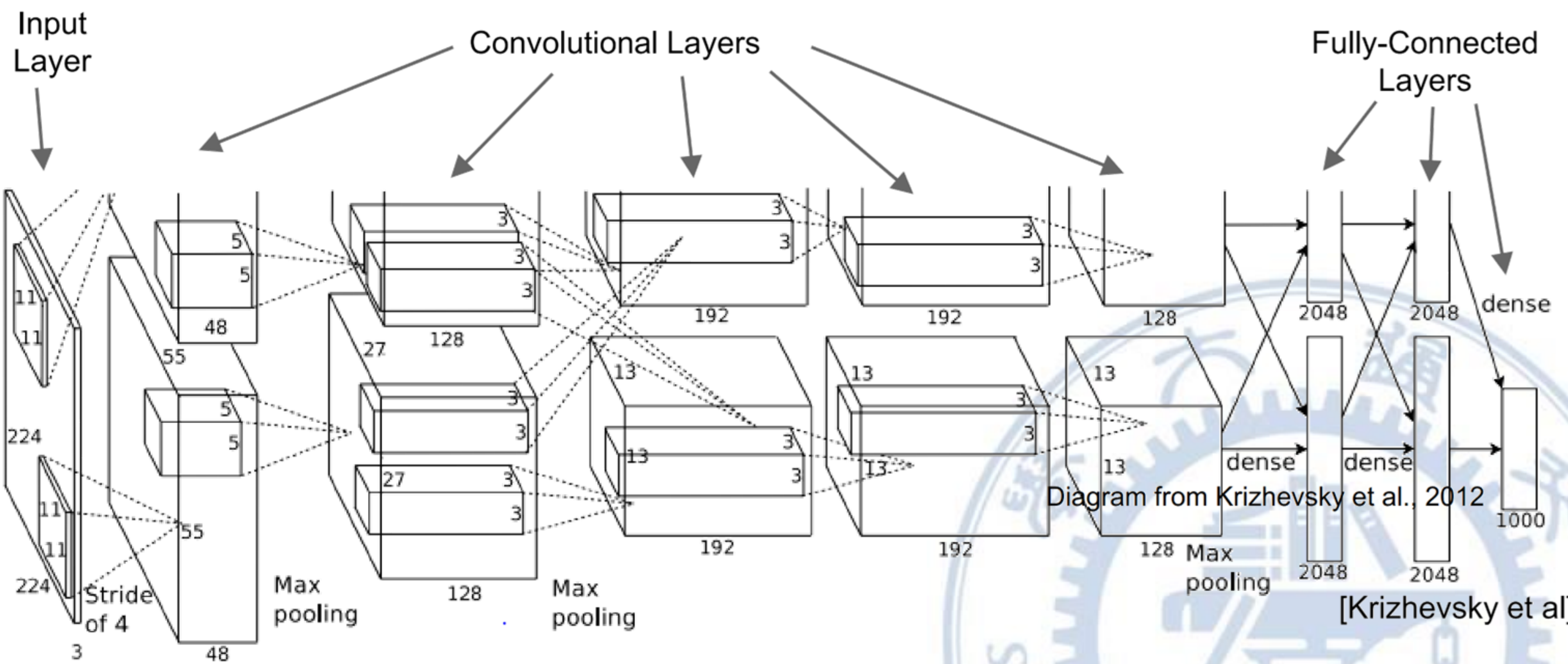




[picture from Simon Thorpe]

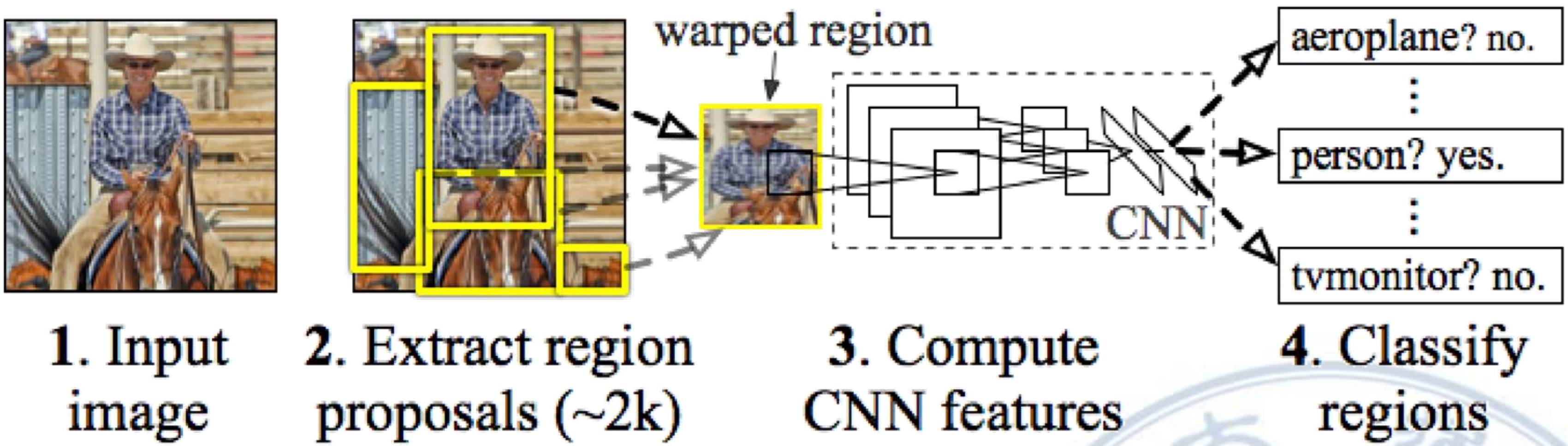
[Gallant & Van Essen]

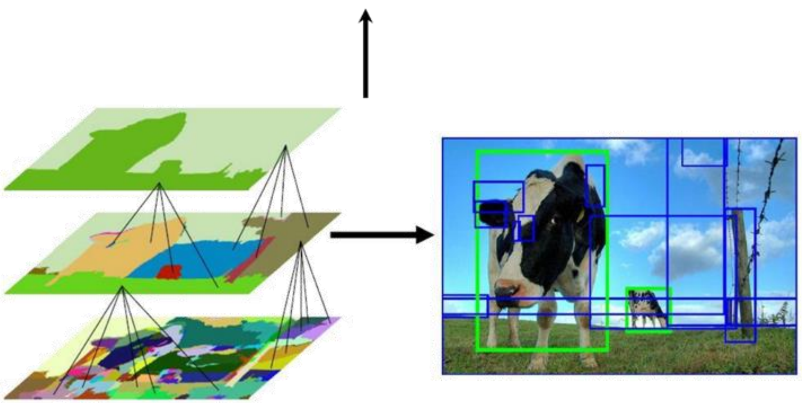
Deep Learning: CNN ILSVRC Architecture



Convolve-Quantize-Pool → [Convolve-Quantize-Pool] → [[Convolve-Quantize-Pool]]→ …

Deep Learning for Object Detection





Top bicycle FPs (AP 62.5%)

Caffe: Open Sourcing Deep Learning

- Convolutional Architecture for Fast Feature Extraction
  - Seamless switching between CPU and GPU
  - Fast computation (2.5ms / image with GPU)
  - Full training and testing capability
  - Reference ImageNet model available
- A framework to support multiple applications:

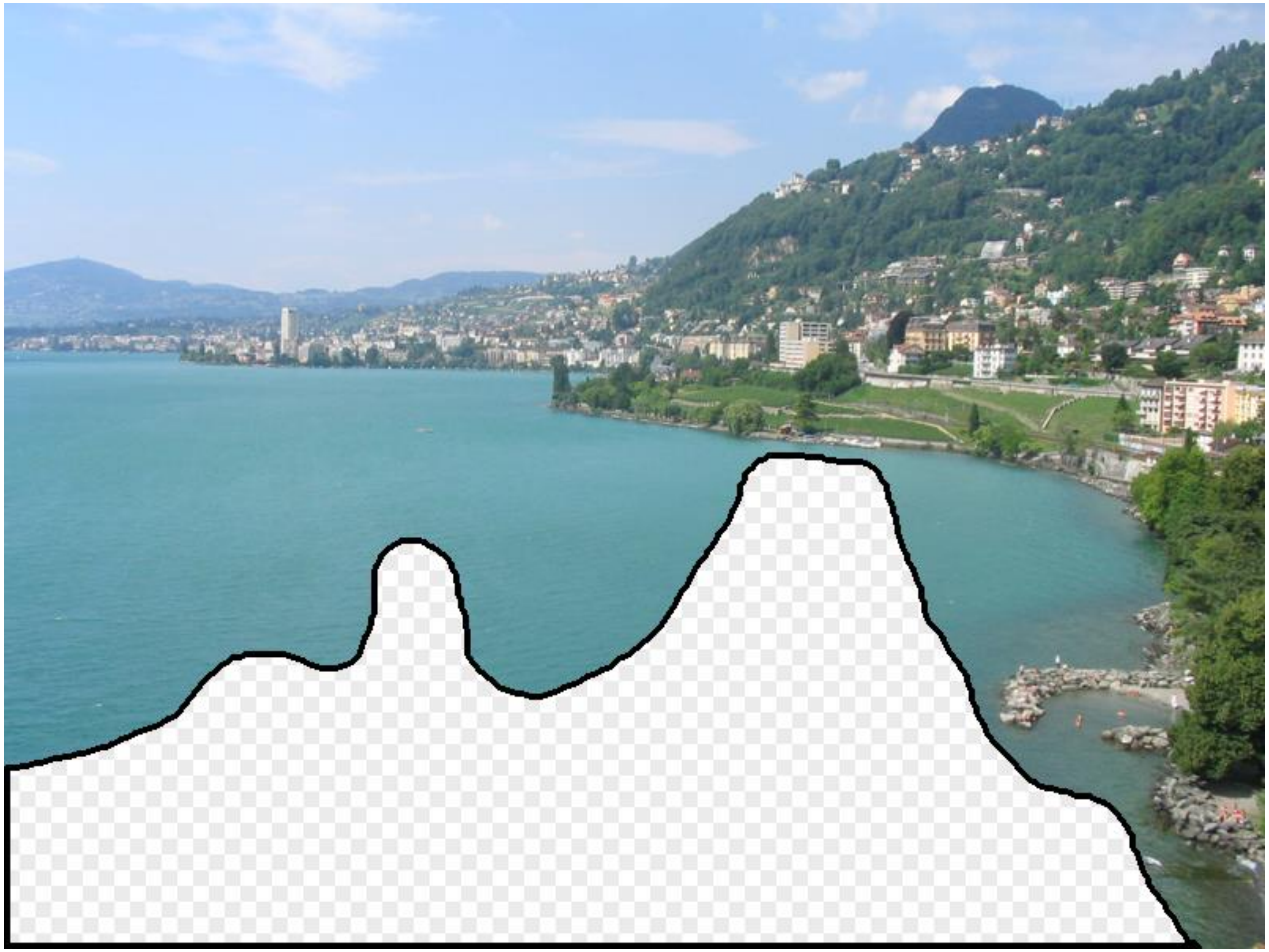
Classification

Embedding

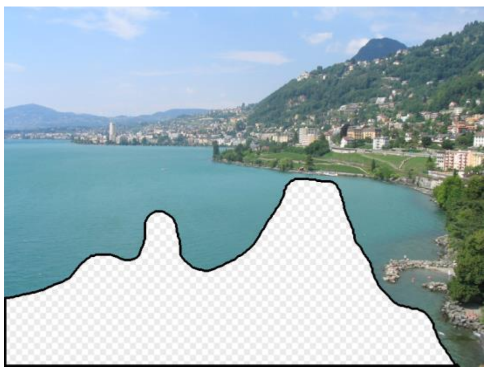
Detection

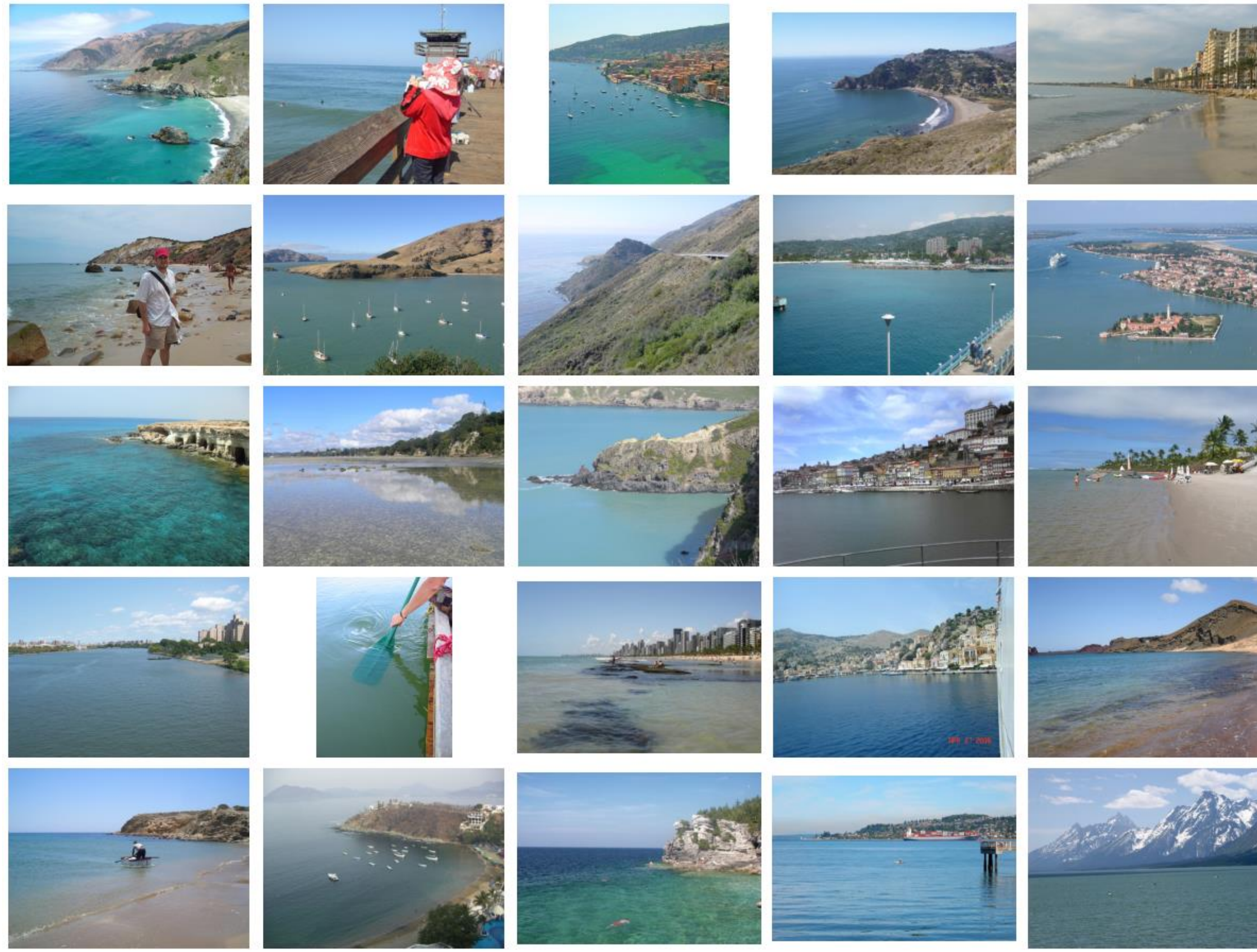
- Main Page
  - http://www.berkeleyvision.org/

Scene Completion



[Hays and Efros. Scene Completion Using Millions of Photographs. SIGGRAPH 2007 and CACM October 2008.]





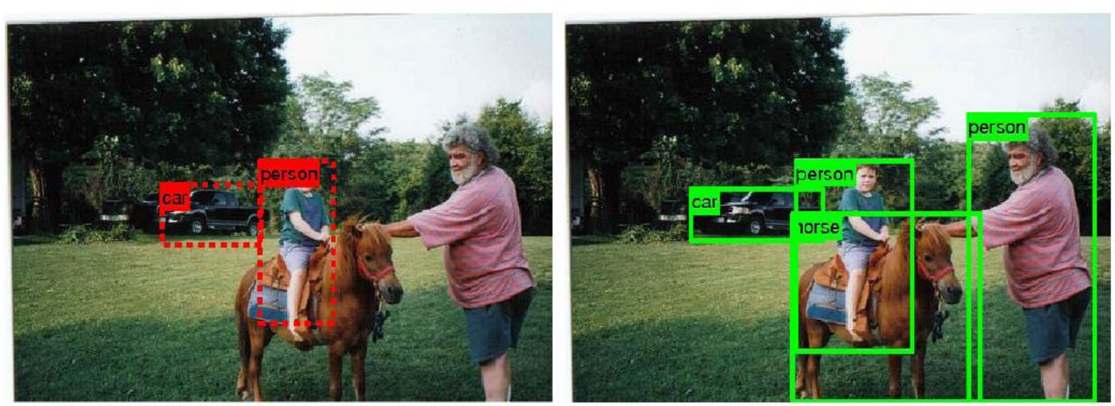
Nearest neighbor scenes from database of 2.3 million photos

Graph cut + Poisson blending

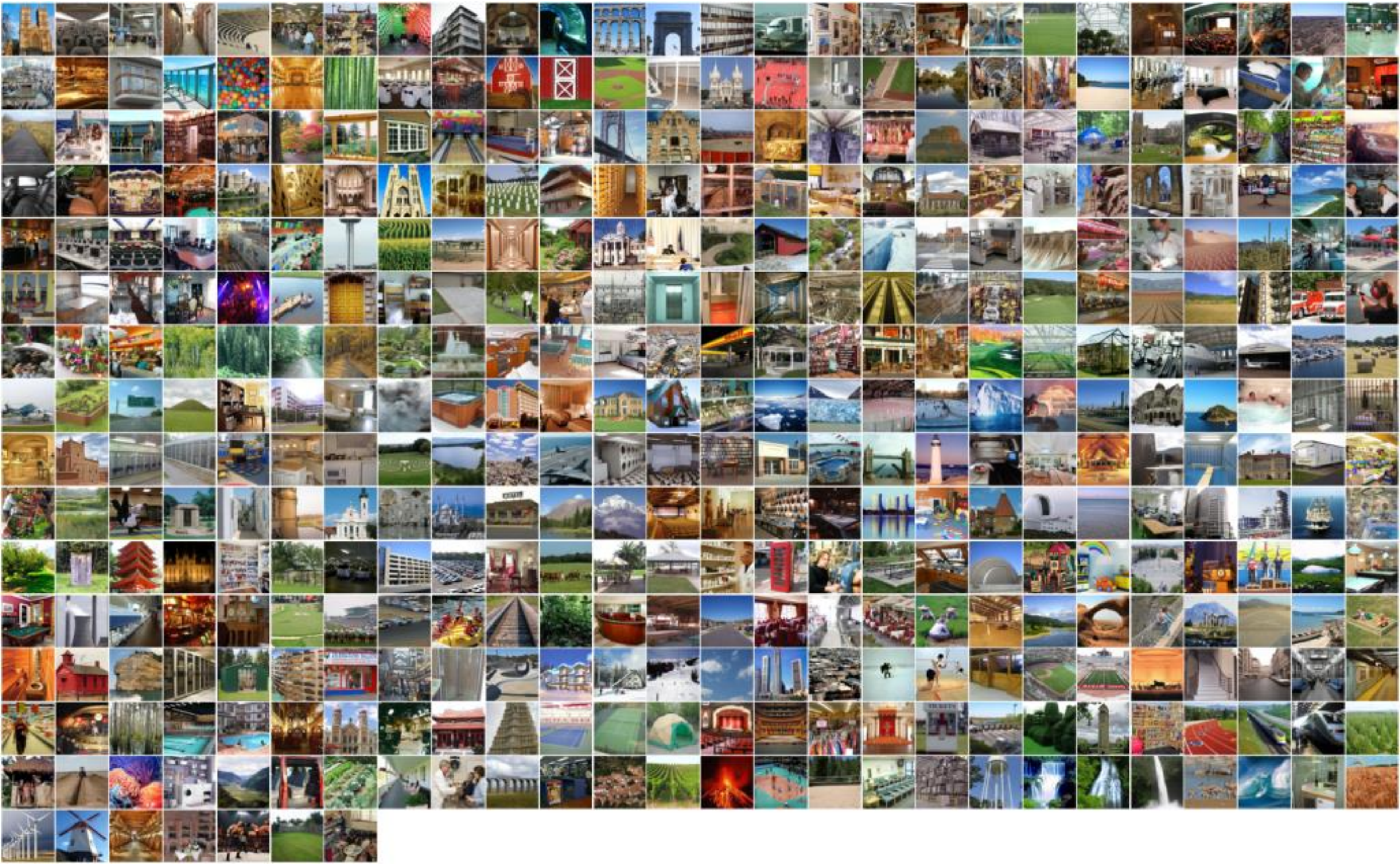
Ongoing Research

IM2GPS: estimating geographic information from a single image

An Empirical Study of Context in Object Detection



Categories of the SUN database



Computer Vision and Nearby Fields

- Computer Graphics: Models to Images
- Comp. Photography: Images to Images
- Computer Vision: Images to Models

Computer Vision

Make computers understand images and video.

What kind of scene?

Where are the cars?

How far is the building?

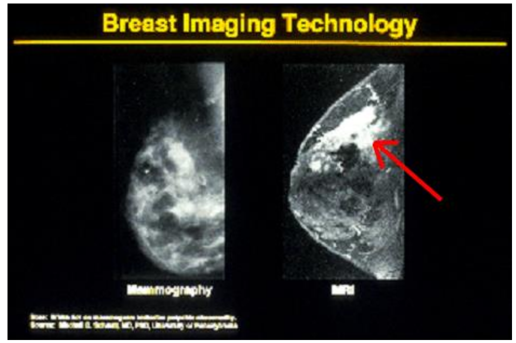
…

Vision is really hard

- Vision is an amazing feat of natural intelligence
  - Visual cortex occupies about 50% of Macaque brain
  - More human brain devoted to vision than anything else

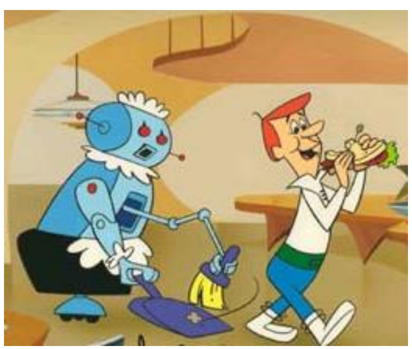
Why computer vision matters

Safety



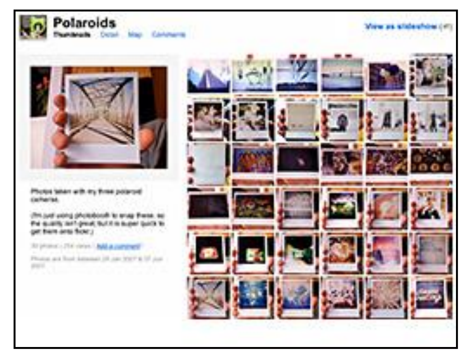
Health

Security



Comfort

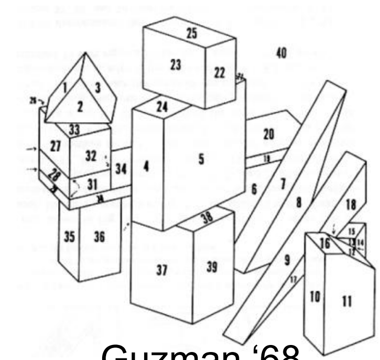
Fun



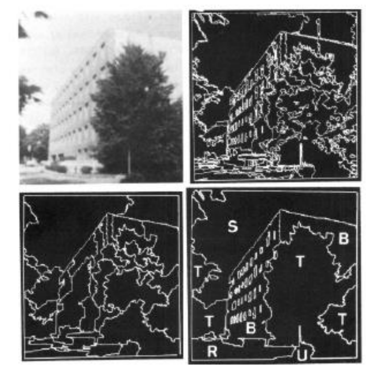
Access

Ridiculously brief history of computer vision

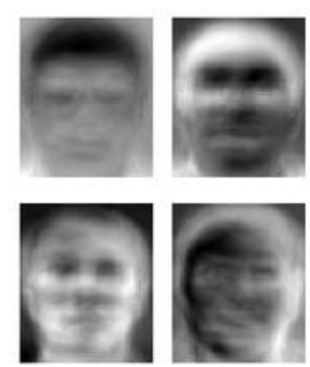
- 1966: Minsky assigns computer vision as an undergrad summer project
- 1960's: interpretation of synthetic worlds
- 1970's: some progress on interpreting selected images
- 1980's: ANNs come and go; shift toward geometry and increased mathematical rigor
- 1990's: face recognition; statistical analysis in vogue
- 2000's: broader recognition; large annotated datasets available; video processing starts



Guzman '68



Ohta Kanade '78

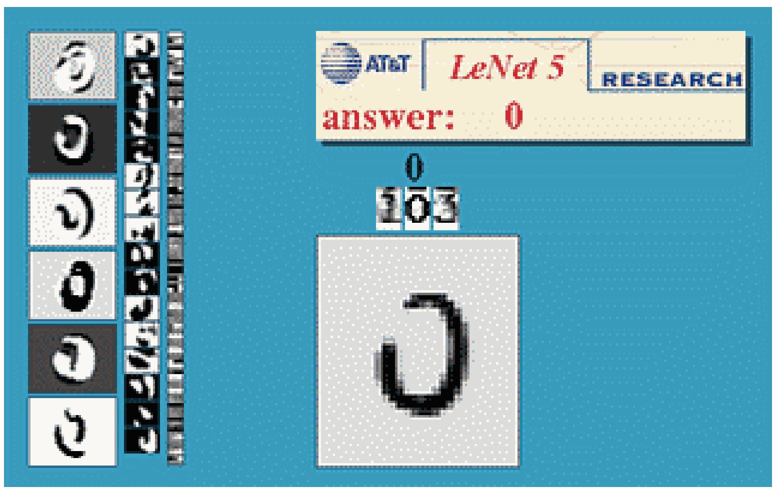


Turk and Pentland '91

Optical character recognition (OCR)

Technology to convert scanned docs to text

- If you have a scanner, it probably came with OCR software



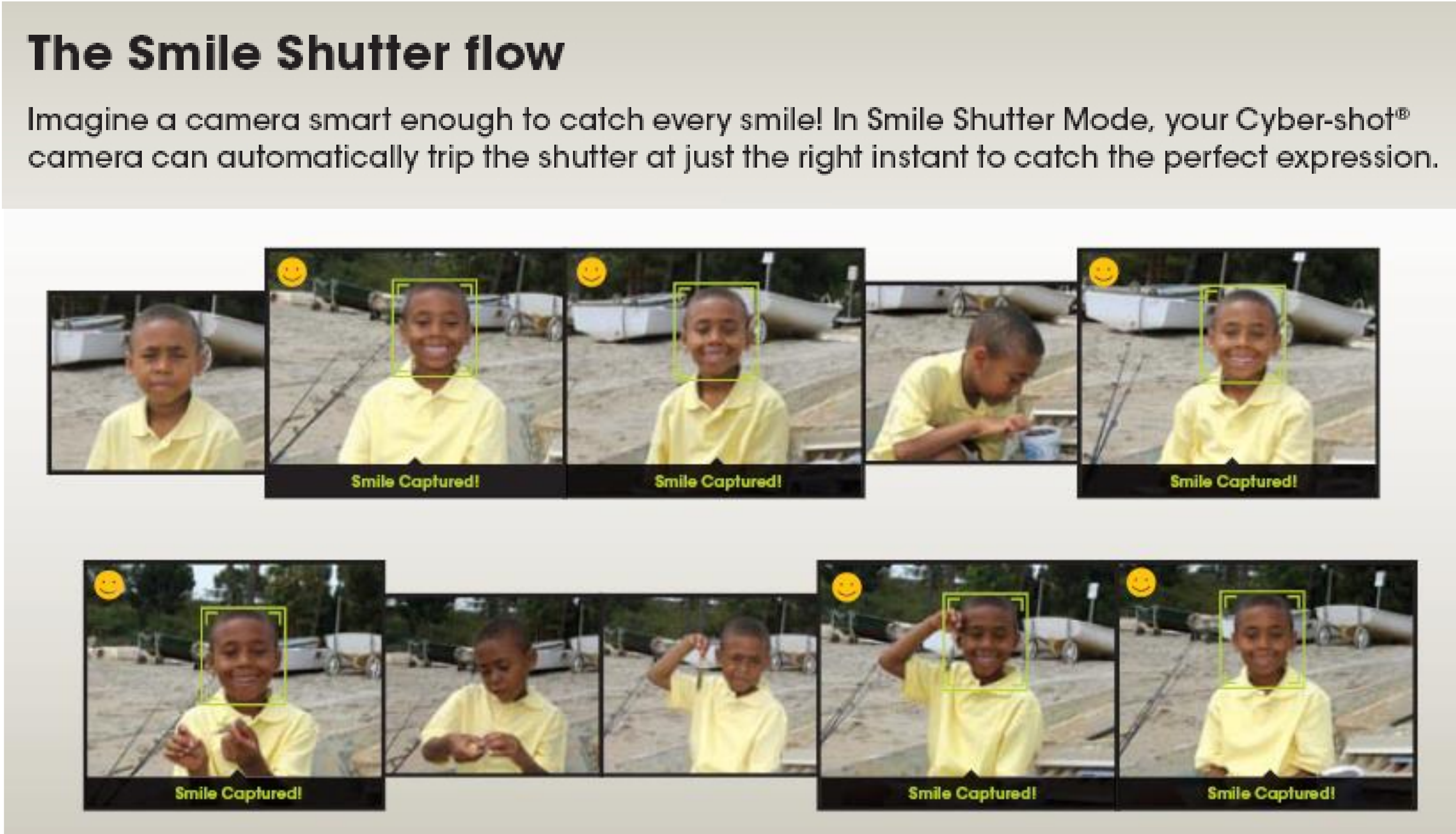
Digit recognition, AT&T labs
http://www.research.att.com/~yann/

License plate readers
http://en.wikipedia.org/wiki/Automatic_number_plate_recognition

Face detection

- Many new digital cameras now detect faces
  - Canon, Sony, Fuji, …

Smile detection



[Sony Cyber-shot® T70 Digital Still Camera](#)

3D from thousands of images

Building Rome in a Day: Agarwal et al. 2009

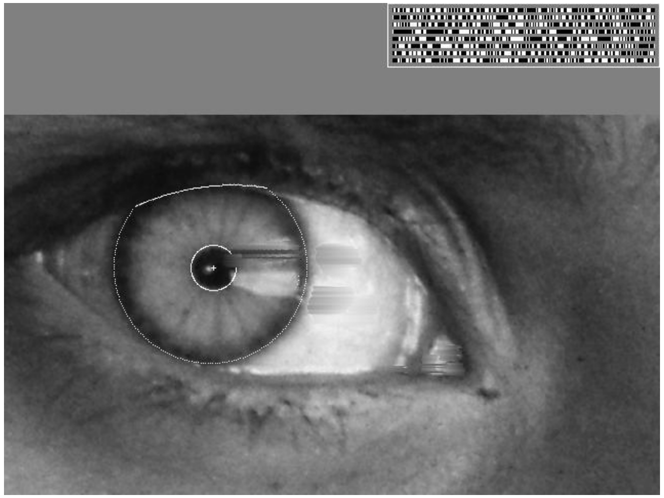
Object recognition (in supermarkets)

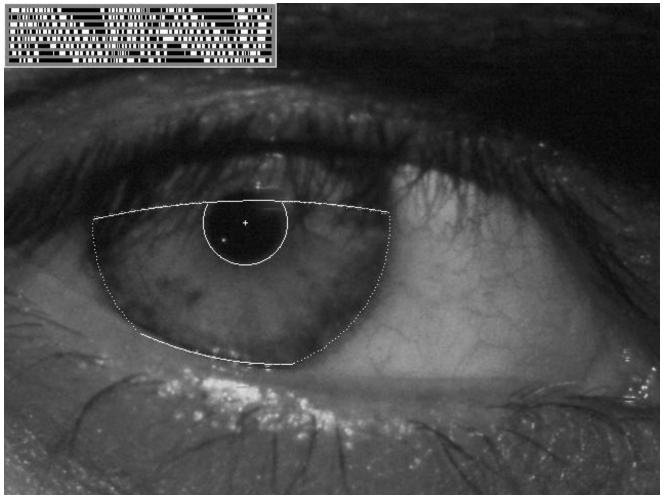
LaneHawk by EvolutionRobotics

"A smart camera is flush-mounted in the checkout lane, continuously watching for items. When an item is detected and recognized, the cashier verifies the quantity of items that were found under the basket, and continues to close the transaction. The item can remain under the basket, and with LaneHawk,you are assured to get paid for it… "

Vision-based biometrics

"*How the Afghan Girl was Identified by Her Iris Patterns*" Read the story
wikipedia





Login without a password…

Fingerprint scanners on
many new laptops,
other devices

Face recognition systems now
beginning to appear more widely
http://www.sensiblevision.com/

Object recognition (in mobile phones)

[Point & Find](), [Nokia]()

[Google Goggles]()

Special effects: shape capture

The Matrix movies, ESC Entertainment, XYZRGB, NRC

Special effects: motion capture

Pirates of the Carribean, Industrial Light and Magic

Sports

Sportvision first down line
Nice explanation on www.howstuffworks.com

http://www.sportvision.com/video.html

Smart cars

Slide content courtesy of Amnon Shashua

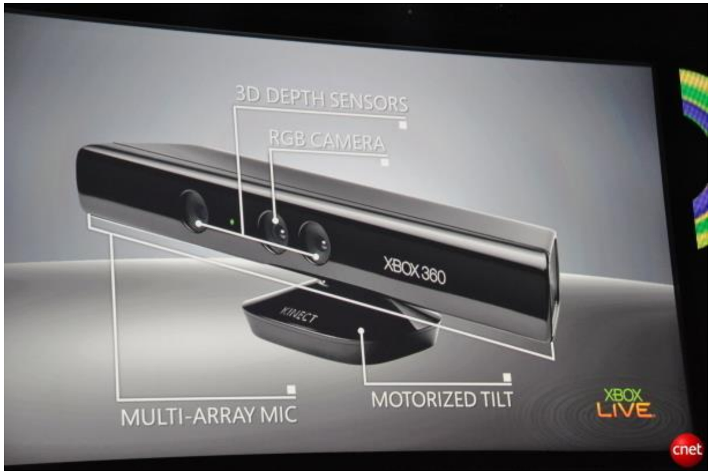
- [Mobileye]
  - Vision systems currently in high-end BMW, GM, Volvo models
  - By 2010: 70% of car manufacturers.

Google cars

http://www.nytimes.com/2010/10/10/science/10google.html?ref=artificialintelligence

Interactive Games: Kinect

- Object Recognition: http://www.youtube.com/watch?feature=iv&v=fQ59dXOo63o
- Mario: http://www.youtube.com/watch?v=8CTJL5lUjHg
- 3D: http://www.youtube.com/watch?v=7QrnwoO1-8A
- Robot: http://www.youtube.com/watch?v=w8BmgtMKFbY



Vision in space

NASA'S Mars Exploration Rover Spirit captured this westward view from atop a low plateau where Spirit spent the closing months of 2007.

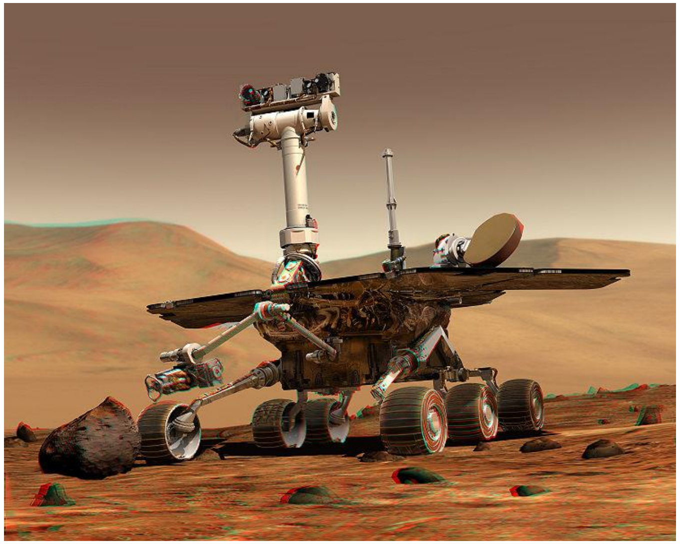
Vision systems (JPL) used for several tasks

- Panorama stitching
- 3D terrain modeling
- Obstacle detection, position tracking
- For more, read "Computer Vision on Mars" by Matthies et al.

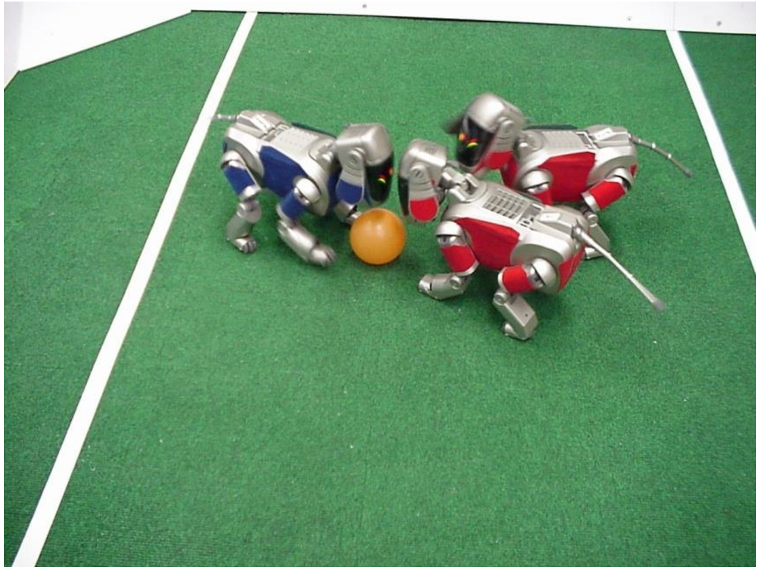
Industrial robots

Vision-guided robots position nut runners on wheels

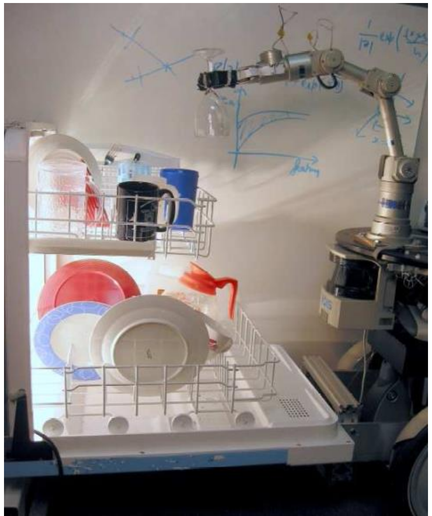
Mobile robots



NASA's Mars Spirit Rover
http://en.wikipedia.org/wiki/Spirit_rover

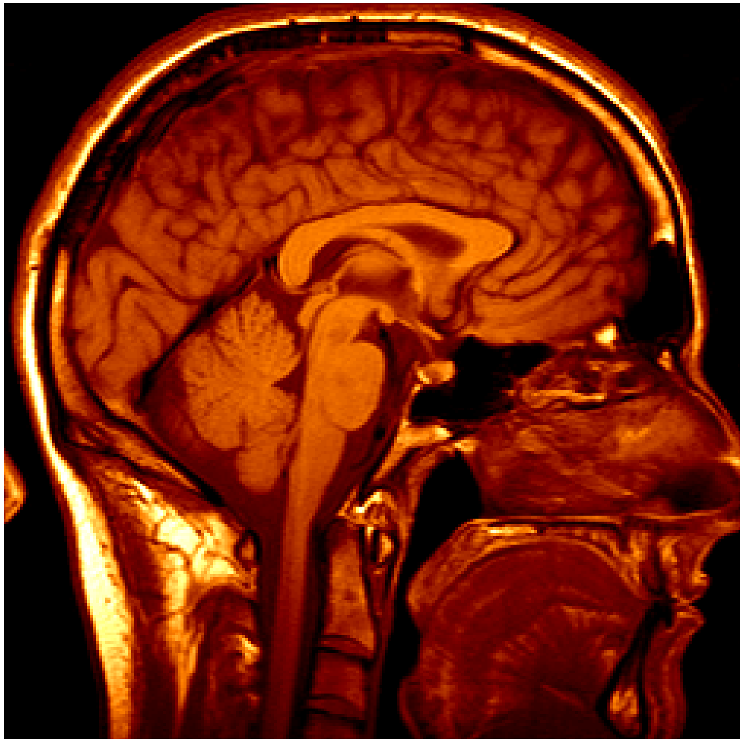


http://www.robocup.org/



Saxena et al. 2008
STAIR at Stanford

Medical imaging



3D imaging
MRI, CT

Image guided surgery
Grimson et al., MIT

Institute of Media,
Information, and Network

Topic

- 1.0 INTRODUCTION
- 1.1 CONTINUOUS-TIME AND DISCRETE-TIME SIGNALS
- 1.2 TRASFORMATION OF INDEPENDENT VARIABLE
- 1.3 EXPONENTIAL AND SINUSOIDAL SIGNALS
- 1.4 THE UNIT IMPULSE AND UNIT STEP FUNCTIONS
- 1.5 Definitions and Representations of Systems
- 1.6 BASIC SYSTEM PROPERTIES

1.1.1 Mathematical Representation of Signals

- Signals are represented mathematically as functions of one or more independent variables

- Described by mathematical expression and waveform

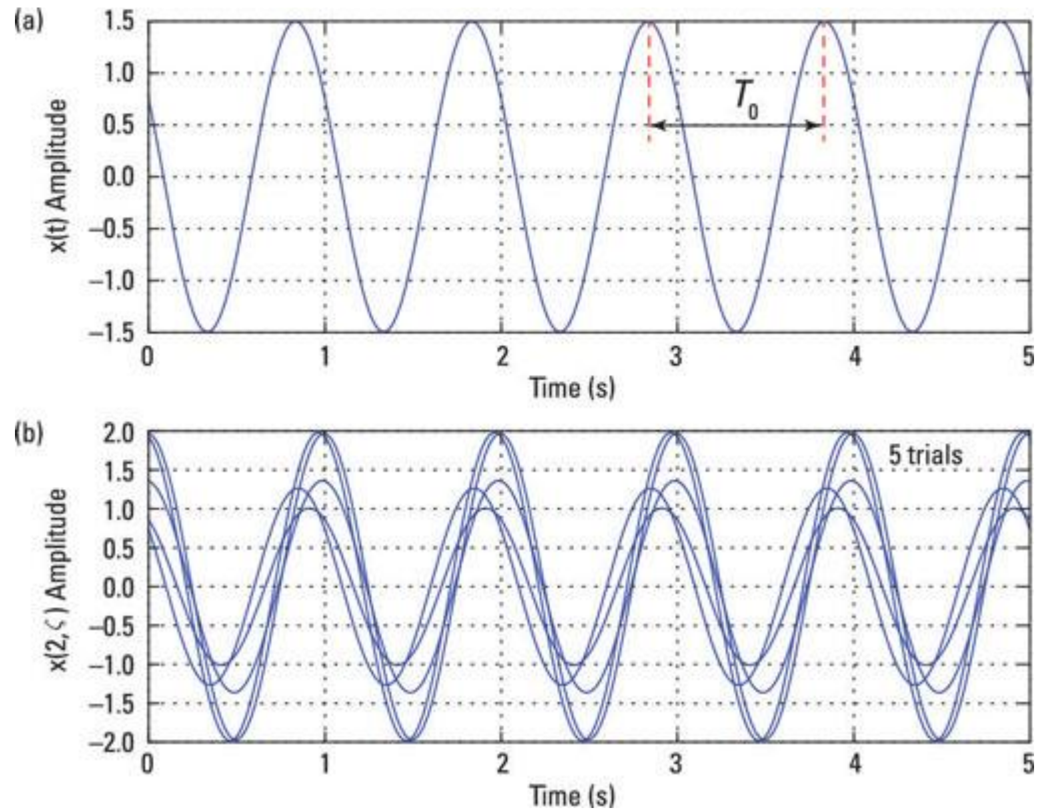
(In this book, we focus our attention on signals involving a single independent variable as time)

1.1.2 Classification of Signals

- Deterministic signal and Random Signal
- Continuous signal and Discrete Signal
- Energy Signal and Power Signal
- Periodic Signal and Non-periodic Signal
- Odd Signal and Even Signal
- Real Signal and Complex Signal
- ....

1.1.2.1 Deterministic signal and Random Signal

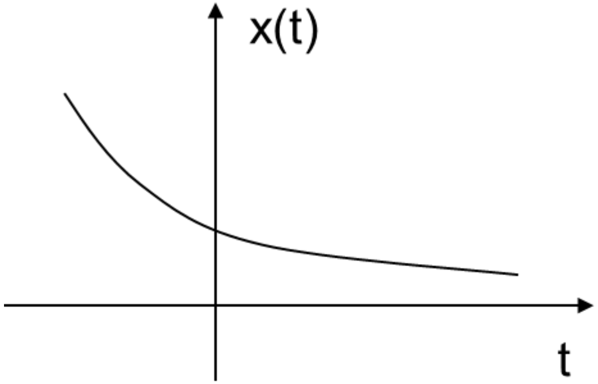
- Deterministic signal
  - Can be described by exact Mathematic expression
  - Given t and get Deterministic result
- Random Signal
  - Can not be described by exact Mathematic expression
  - Given t and get random result



1.1.2.2 Continuous-Time (CT) and Discrete-Time (DT) Signals:

- Continuous-Time (CT) Signals: x(t)
  - Independent variable (t) is continuous
  - The signal is defined for a continuum of values of the independent variable (t)

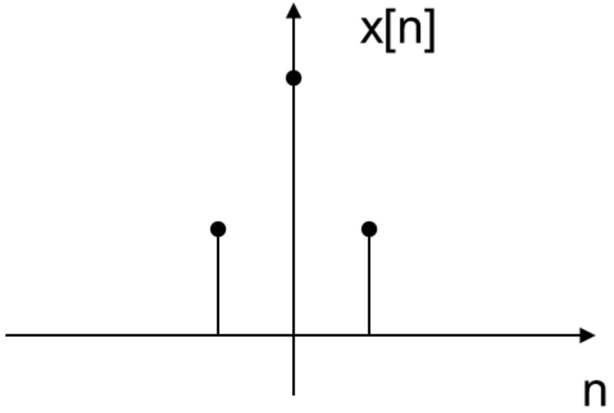
$$example: \quad x(t) = 2e^{-t}$$



1.1.2.2 Continuous-Time (CT) and Discrete-Time (DT) Signals:

- Discrete-Time (DT) Signals/Sequences: x[n]
  - Independent variable (n) takes on only a discrete set of values, in this course, a set of integer values only
  - Signal is defined only at discrete times

$$example: \quad x[n] = \begin{cases} 2, & n = -1 \\ 4, & n = 0 \\ 2, & n = 1 \\ 0, & others \end{cases}$$



1.1.2.3 Time-Limited and Power-Limited Signals

Power and energy in a physical system

- Instantaneous power $$P(t) = v(t)i(t) = \frac{1}{R}|v(t)|^2$$
- Total energy over time interval $[t_1, t_2]$ $$\int_{t_1}^{t_2} p(t)dt = \frac{1}{R}\int_{t_1}^{t_2} |v(t)|^2 dt$$
- Average power over time interval $[t_1, t_2]$ $$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2 - t_1}\frac{1}{R}\int_{t_1}^{t_2} |v(t)|^2 dt$$

1.1.2.3 Time-Limited and Power-Limited Signals

Power and energy definitions in the course

- Total Energy

$$E \overset{\Delta}{=} \int_{t_1}^{t_2} |x(t)|^2 \, dt \qquad\qquad E \overset{\Delta}{=} \sum_{n=n_1}^{n_2} |x[n]|^2$$

- Average Power

$$P \overset{\Delta}{=} \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt \qquad\qquad P \overset{\Delta}{=} \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

1.1.2.3 Time-Limited and Power-Limited Signals

Power and energy definitions over an infinite interval

- Total Energy

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt \qquad E_\infty = \lim_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2$$

- Average Power

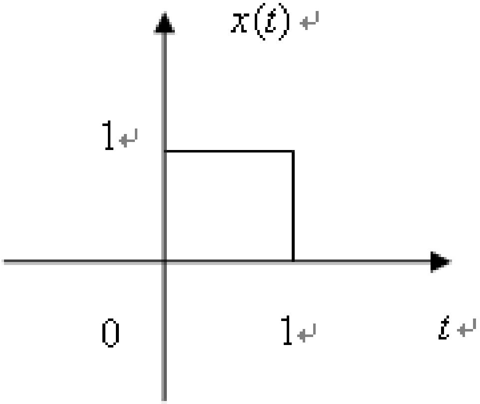
$$P_\infty = \frac{1}{2T} \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt \qquad P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

1.1.2.3 Time-Limited and Power-Limited Signals

- Finite-Energy Signal

$E_\infty < \infty \qquad P_\infty = 0$

example:



- Finite-Average Power Signal

$P_\infty < \infty \qquad E_\infty = \infty$

example: $x[n]=4$

1.1.2.4 Periodic and Non-Periodic Signals

For continuous-time signals

- Definition:

  If $x(t) = x(t+T)$ for all values of t, $x(t)$ is periodic Then $x(t) = x(t+mT)$ for all t and any integral m

  - Fundamental Period: the smallest positive value of satisfying $x(t) = x(t+T)$ for all t

If the signal is constant, the fundamental period ?

1.1.2.4 Periodic and Aperiodic Signals

For discrete-time signals

- Definition:

  If $x[n] = x[n+N]$ for all values of n, $x[n]$ is periodic Then $x[n] = x[n+mN]$ for all n and any integral m

  - Fundamental Period: the smallest positive value of satisfying $x[n] = x[n+N]$ for all n

If the signal is constant,
the fundamental period ?

1.1.2.5 Even and Odd Signals

- Definition:

  $x(t)$ or $x[n]$ is even if it is identical to its time-reversed counterpart

$$x(t) = x(-t) \qquad\qquad x[n] = x[-n]$$

  Similarly $x(t)$ or $x[n]$ is odd if

$$x(t) = -x(-t) \qquad\qquad x[n] = -x[-n]$$

For odd signal $x(t)$, can one determine $x(0)$?

1.1.2.5 Even and Odd Signals

- Even-odd decomposition of a signal

$$x(t) = E_v\{x(t)\} + O_d\{x(t)\}$$

Even part   Odd part

$$E_V\{x(t)\} = \frac{1}{2}[x(t) + x(-t)] \qquad O_d\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

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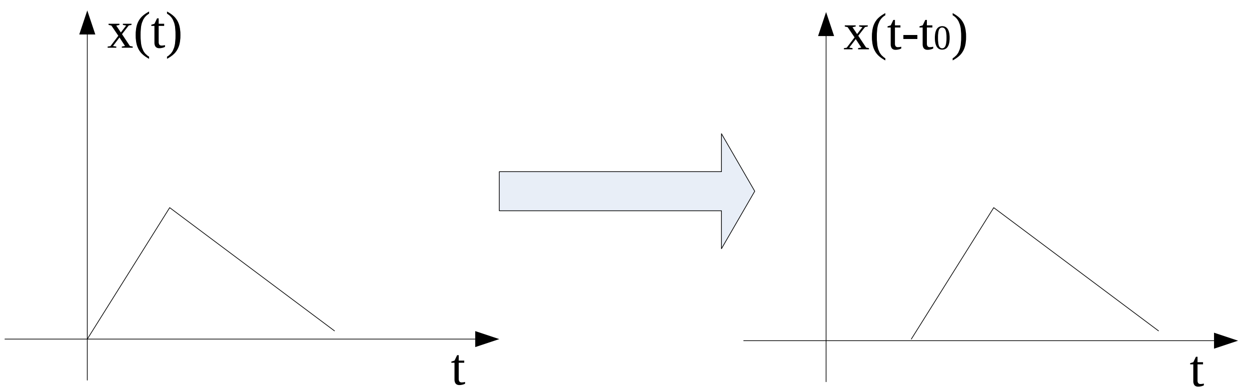
1.2.1 Time Shift

$$x(t) \to x(t - t_0) \qquad x[n] \to x[n - n_0]$$



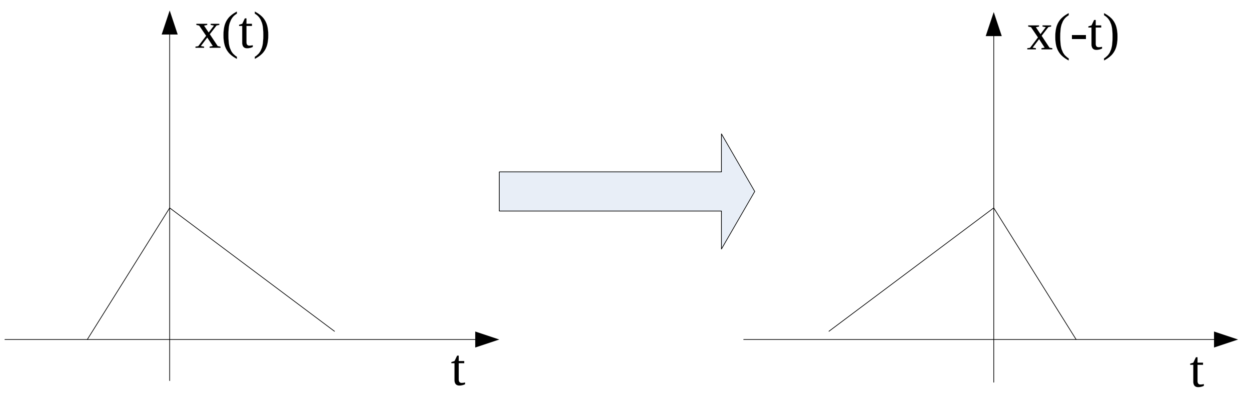
e.g.: Radar, Sonar, Radio propagations

Notes: Each point in x(t)/x[n] occurs at a later/early time in $x(t-t_0)/x[n-n_0]$, when t_0/n_0 is positive/negative, i.e.

- $x(t-t_0)/x[n-n_0]$ is the delayed version of x(t)/x[n], for $t_0/n_0 > 0$
- $x(t-t_0)/x[n-n_0]$ is the advanced version of x(t)/x[n], for $t_0/n_0 < 0$

1.2.2 Time Reversal

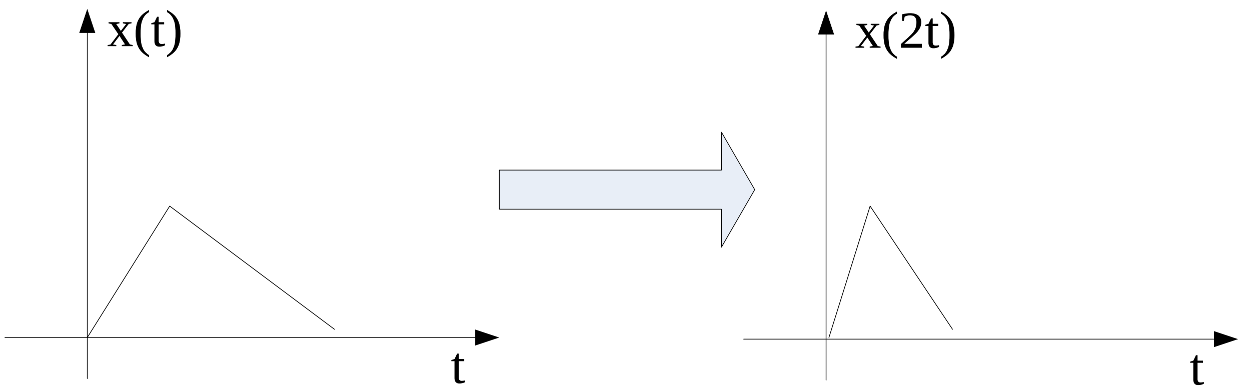
$$x(t) \rightarrow x(-t) \qquad\qquad x[n] \rightarrow x[-n]$$



e.g.： tape recording played backward

1.2.3 Time Scaling

$$x(t) \rightarrow x(\alpha t) \qquad x[n] \rightarrow x[\alpha n]$$



E.g. tape recording played:

- fast forward $\alpha > 1$
- slow forward $0 < \alpha < 1$
- slow backward $-1 < \alpha < 0$
- fast backward $\alpha < -1$

Notes:

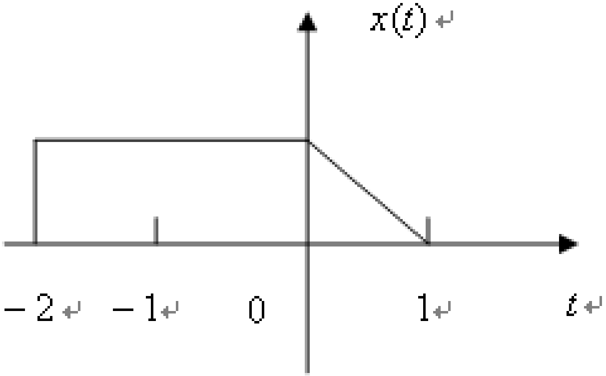
$|\alpha| > 1$ —Compression

$|\alpha| < 1$ —Extension

1.2.4 A General Transform of the Independent Variable

$$x(t) \to x(\alpha t + \beta) \qquad x[n] \to x[\alpha n + \beta]$$

$$example: \quad x(t) \to x(-3t - 2)$$

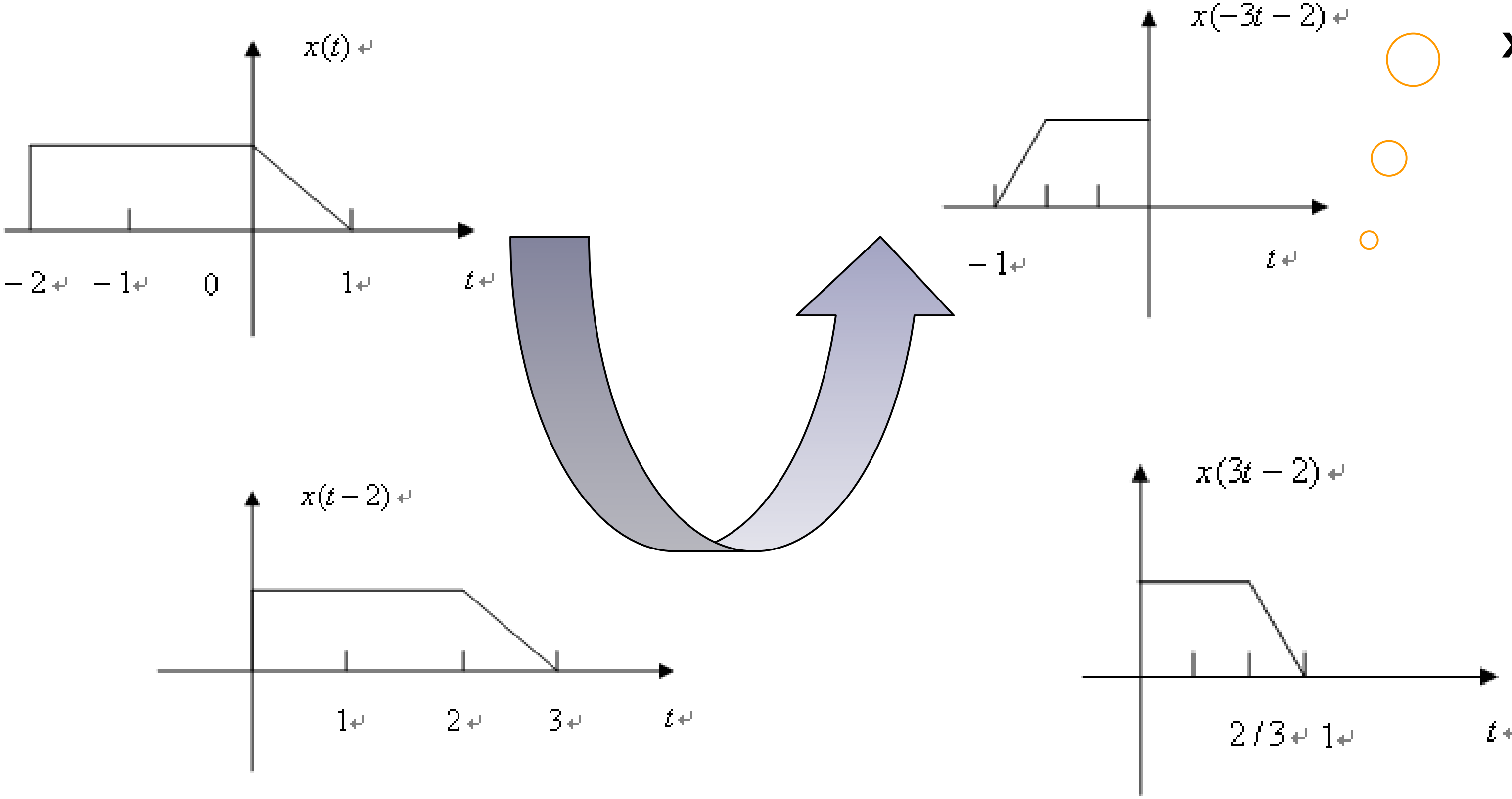


Rule:

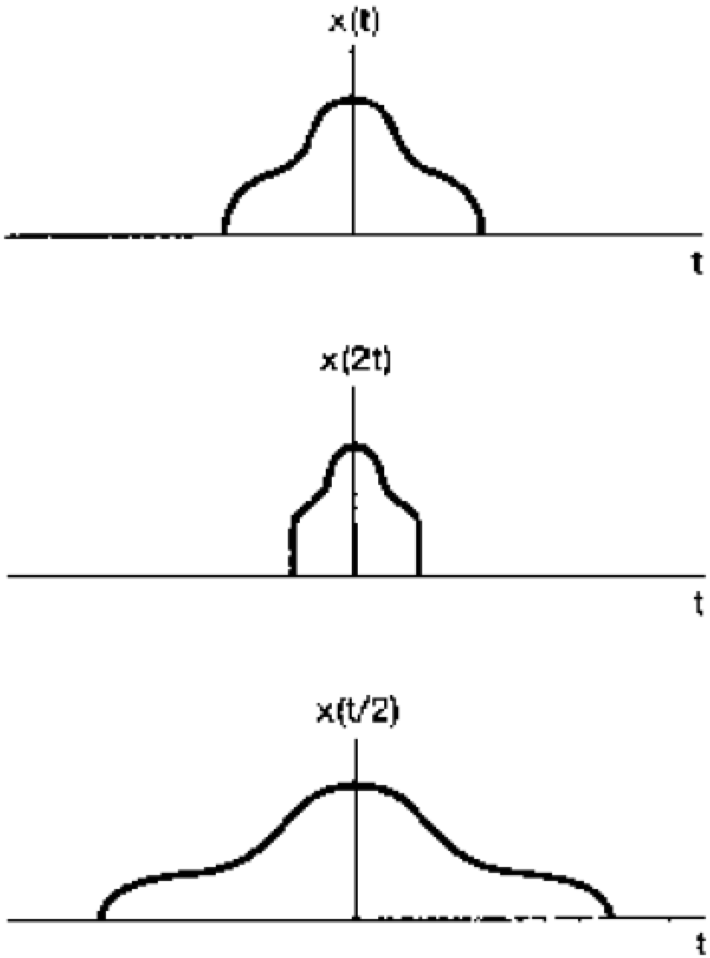
1. **time shift first**
2. **then reflection(time reversal) and time scaling**

Question: what happens if shifting after scaling/reflection

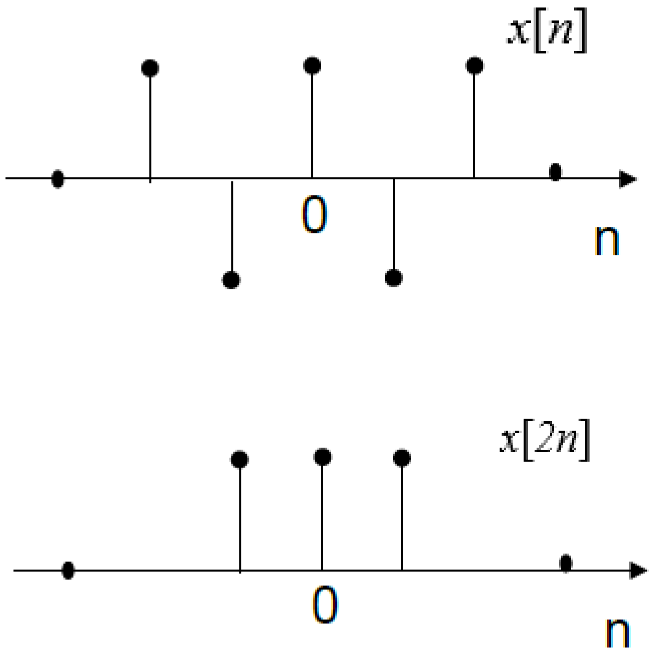
x(-3(t+2/3))



Example: x(t)、x(2t)、x(t/2)



Example: x[n]、x[2n]



Topic

- 1.0 INTRODUCTION
- 1.1 CONTINUOUS-TIME AND DISCRETE-TIME SIGNALS
- 1.2 TRASFORMATION OF INDEPENDENT VARIABLE
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1.3.1 Continuous-Time Complex Exponentials Signals and Sinusoidal Signals

$$x(t) = C \cdot e^{\alpha t}$$

Where C and α are complex numbers

- Real Exponential Signals: when C and α are real numbers, e.g. $x(t) = e^{2t}$
  - growing exponential, when $\alpha > 0$
  - decaying exponential, when $\alpha < 0$
  - constant $\alpha = 0$

- Periodic Complex Exponential and Sinusoidal Signals:
  when C is real, α is purely imaginary, e.g.
  then the **fundamental period** $T_0 = 2\pi/\omega_0$ [s] , angular frequency ω_0[rad/s], and frequency $f_0 = \frac{\omega_0}{2\pi} = 1/T_0$[Hz]
  Unless noted otherwise, in this course, we always call ωo frequency

Euler's Relation

$$e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$$

$$\cos\omega_0 t = \frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t}) \quad \sin\omega_0 t = \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})$$

- Important periodicity property :
  - 1) the larger the magnitude of ω_0 , the higher the oscillation in the signal
  - 2) the signal $x(t)$ is periodic for any value of ω_0

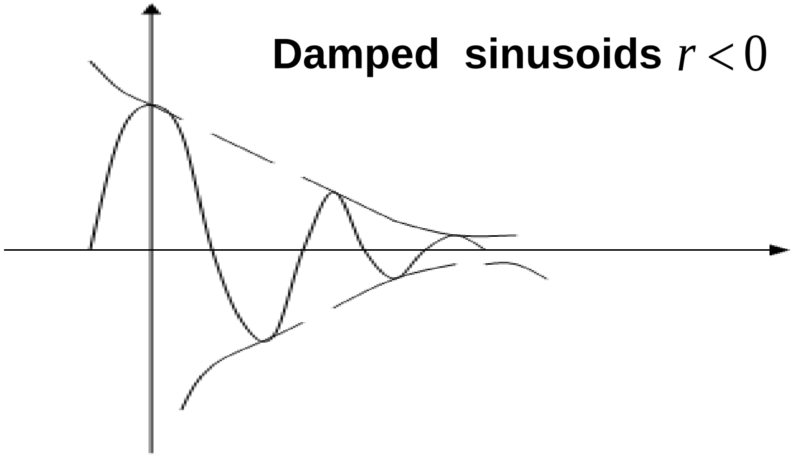
- A general representation, when C and α are complex numbers, denoted as $C=|C|e^{j\theta}$, $\alpha = r + j\omega_0$, then

$$x(t)=|c|\cdot e^{j\theta}\cdot e^{(r+j\omega_0)t}=|c|\cdot e^{rt}\cdot e^{j(\omega_0 t+\theta)}$$

$|c|\cdot e^{rt}$ is the envelop of the waveform

ω_0 is the oscillation frequency

Example of real part of $x(t)$



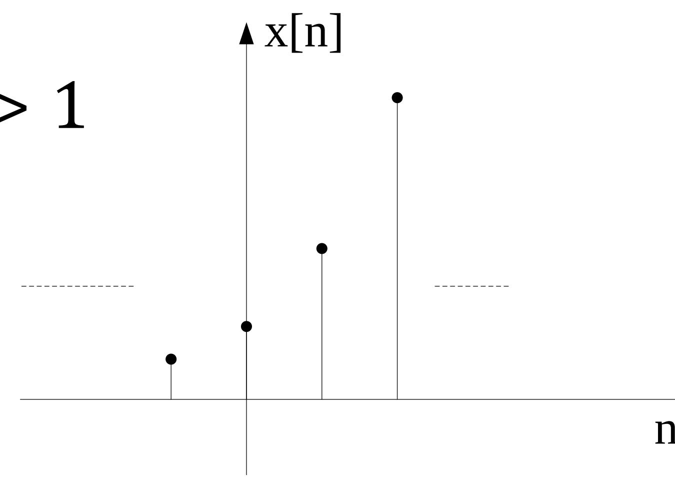
1.3.2 Discrete-Time Complex Exponentials Signals and Sinusoidal Signals

$$x[n] = C \cdot \alpha^n$$

Where C and α are complex numbers

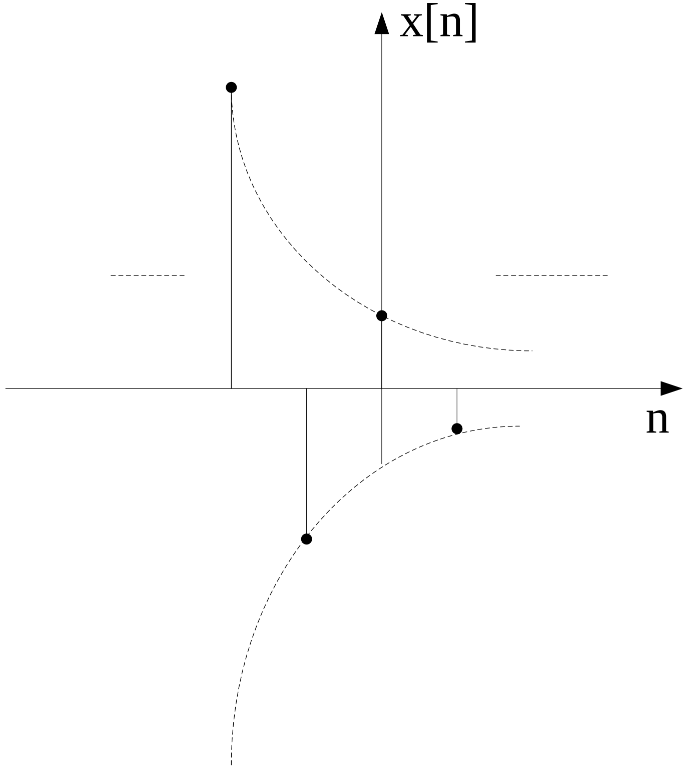
- Real Exponential Signals: when C and α are real numbers
  - e.g. growing function, when $|\alpha| > 1$

$$x[n] = 2^n$$



- decaying function, when $0 < |\alpha| < 1$

$$x[n] = (-1/2)^n$$



- constant, when $|\alpha| = 1$
- alternates in set $\{-C, C\}$, when $|\alpha| = -1$

- Complex Exponential and Sinusoidal Signals: when C is real, α is a point on the unit circle, e.g.

$$x[n] = e^{j\omega_0 n} \quad \text{or} \quad x[n] = A\cos(\omega_0 n + \phi), A\sin(\omega_0 n + \phi)$$

Its periodicity property? Similar to that of continuous-time signals?

- A general representation, when C , α are complex numbers, denoted as $\mathrm{C} = |\mathrm{C}|e^{j\theta}$, $\alpha = re^{j\omega_0}$,then

$$x[n] = |c| \cdot e^{j\theta} \cdot r^n e^{j\omega_0 \mathrm{n}} = |c| \cdot r^{\mathrm{n}} \cdot e^{j(\omega_0 \mathrm{n} + \theta)}$$

- $|c| \cdot r^n$ is the envelop of the waveform

- **Periodicity Property of Discrete-time Complex Exponentials** $x[n] = e^{j\omega_0 n}$
  - a) recall the definition of the periodic discrete-time signal $x[n] = x[n+N]$ for all n
  - b)if it is periodic, there exists a positive integer N, which satisfies $e^{j\omega_0 n} = e^{j\omega_0 (n+N)} = e^{j\omega_0 n} e^{j\omega_0 N}$ so, it requires $e^{j\omega_0 N} = 1$, i.e. $\omega_0 N = 2\pi m$
  - If there exists an integer satisfying that $2\pi m/\omega_0$ is an integer, i.e. $2\pi/\omega_0$ is rational number , $x[n]$ is periodic with fundamental period of $\mathrm{N} = 2\pi m/\omega_0$, where N, m are integers without any factors in common.

    otherwise, $x[n]$ is aperiodic.

Different from that of continuous exponentials

- Another difference from that of CT exponentials

since $e^{j\omega_0 n} = e^{j(\omega_0+2\pi m)n}$ for any integer m
the signal is fully defined within a frequency interval of length 2π : $((2m-1)\pi, \quad (2m+1)\pi]$, for any integer m

Distinctive signals for different ω_0 within any 2π region, i.e.
$((2m-1)\pi, \quad (2m+1)\pi]$ for any integer m

Without loss of generalization, for $\omega_0 \in (-\pi, \quad \pi]$, the rate of oscillation in the signal $e^{j\omega_0 n}$ increases with $|\omega_0|$ increases from 0 to π

Important for discrete-time filter design!

- Comparison of Periodic Properties of CT and DT Complex Exponentials/ Sinusoids

$x(t) = e^{j\omega_0 t}$	$x[n] = e^{j\omega_0 n}$
Distinct signals for distinct value of ω_0	Identical signals for values of ω_0 separated by multiples of 2π
Periodic for any choice of ω_0	Periodic only of $\omega_0 = 2\pi m / N$ for some integers $N > 0$ and m
Fundamental angular frequency ω_0	Fundamental angular frequency ω_0 / m, if m and N do not have any factors in common
Fundamental period $\frac{2\pi}{\omega_0}$	Fundamental period $m\left(\frac{2\pi}{\omega_0}\right)$

Topic

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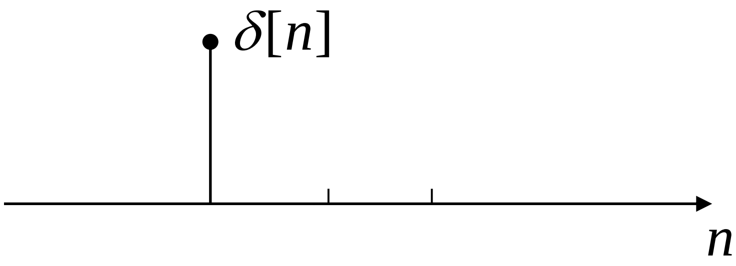
- 1.6 BASIC SYSTEM PROPERTIES

1.4.1 Discrete-Time Unit Impulse and Unit Step Sequences

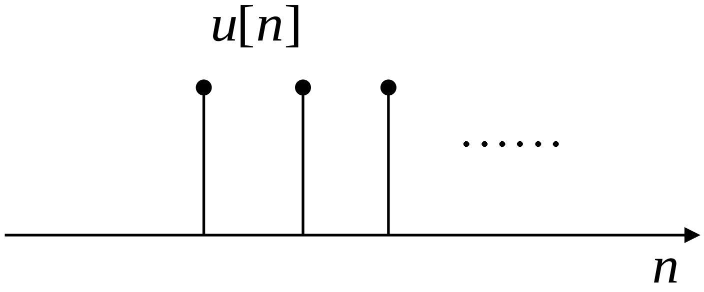
- Unit Impulse Sequence

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$



- Unit Step Sequence

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$



- Relationship

$$\delta[n] = u[n] - u[n-1]$$ —1st difference

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \ / \ u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$ —running sum

- Sampling Property

$$x[n] \cdot \delta[n] = x[0] \cdot \delta[n]$$

$$x[n] \cdot \delta[n-n_0] = x[n_0] \cdot \delta[n-n_0]$$

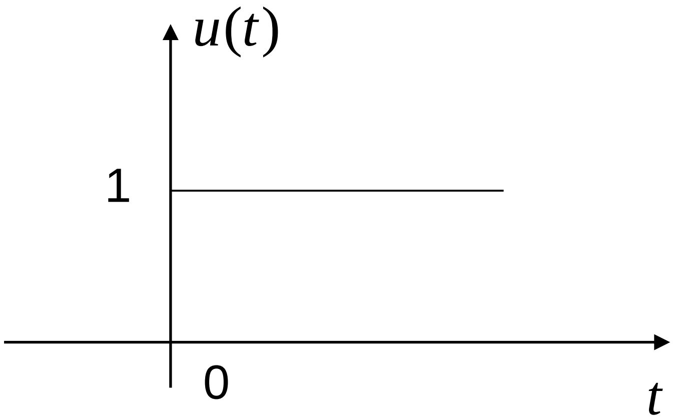
- Signal representation by means of a series of delayed unit samples

$$x[n] = \sum_{k} x[k] \cdot \delta[n-k]$$

1.4.2 Continuous-Time Unit Step and Unit Impulse Functions

- Unit Step Function

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



Notes: $u(t)$ is undefined at $t = 0$

Can we find counterpart of the unit impulse function in CT domain as that in DT domain ?

$$\delta[n] = u[n] - u[n-1]$$ —1st difference

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \;/\; u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$ —running sum

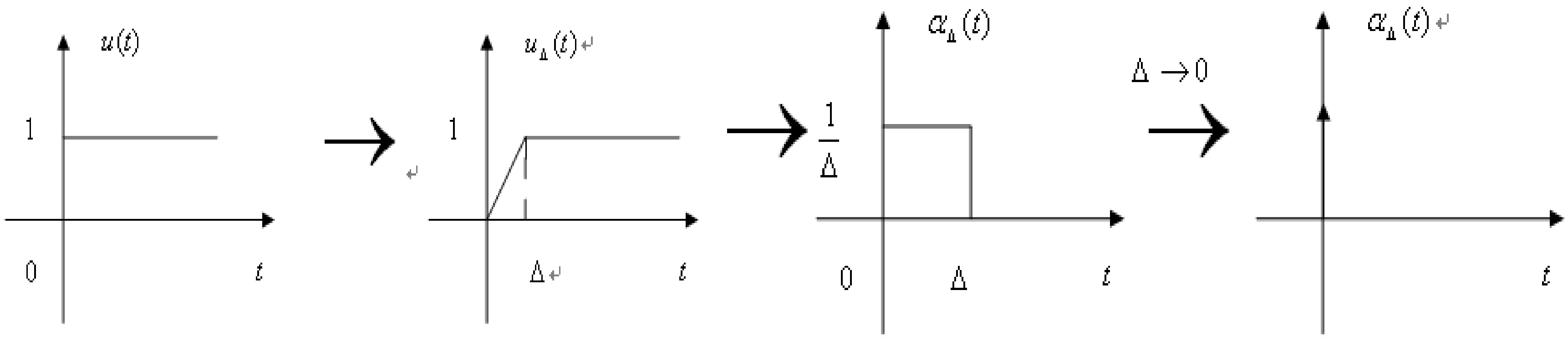
Does it exist $\delta(t)$ satisfying the following relationship

$$\delta(t) = \frac{du(t)}{dt}$$ —1st derivative

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$ —running sum

?

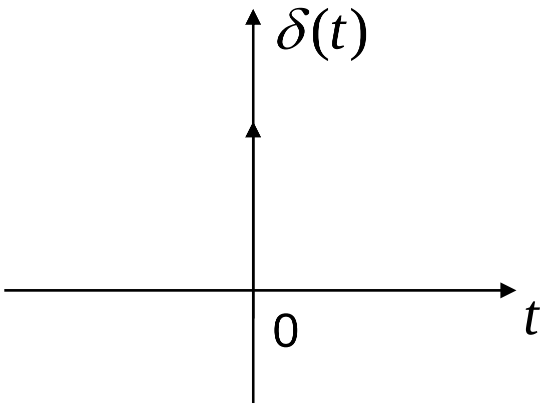
- Unit Impulse Function
  - Since $u(t)$ is undefined at $t = 0$, formally it is not differentiable, then define an approximation to the unit step $u_\Delta(t)$,which rises from 0 to 1 in a very short interval Δ
  - So $\delta_\Delta(t) = \dfrac{d(u_\Delta(t))}{dt}$
  - And $\delta(t) = \lim\limits_{\Delta \to 0} \delta_\Delta(t)$



Notes: the amplitude of the signal $\delta(t)$ at $t = 0$ is infinite, but with unit integral from $-\infty$ to ∞ , i.e. from $0^-$to 0^+

- Unit Impulse Function
  - Dirac Definition

$$\begin{cases} \int_{-\infty}^{\infty} \delta(t)dt = 1 \\ \delta(t) = 0 \quad t \neq 0 \end{cases}$$



Notes: the amplitude of the signal $\delta(t)$ at $t = 0$ is infinite, but with unit integral from $-\infty$ to ∞, i.e. from 0^- to 0^+

  - We also call such functions as singularity function or generalized functions, for more information, please refer to mathematic references

- Relationship

$$\delta(t) = \frac{du(t)}{dt}$$ —1st derivative

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$$ —running sum

- Sampling Property

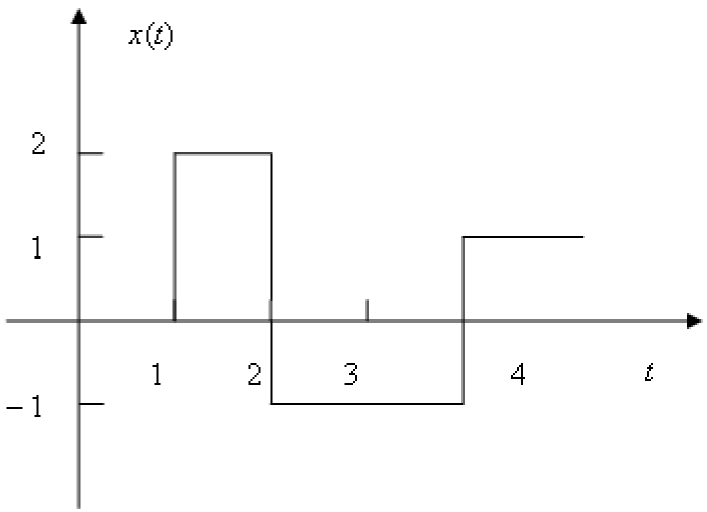
$$x(t)\cdot\delta(t) = x(0)\cdot\delta(t)$$

$$x(t)\cdot\delta(t-t_0) = x(t_0)\cdot\delta(t-t_0)$$

Can we represent x(t) by using a series of unit samples as that for DT signal?

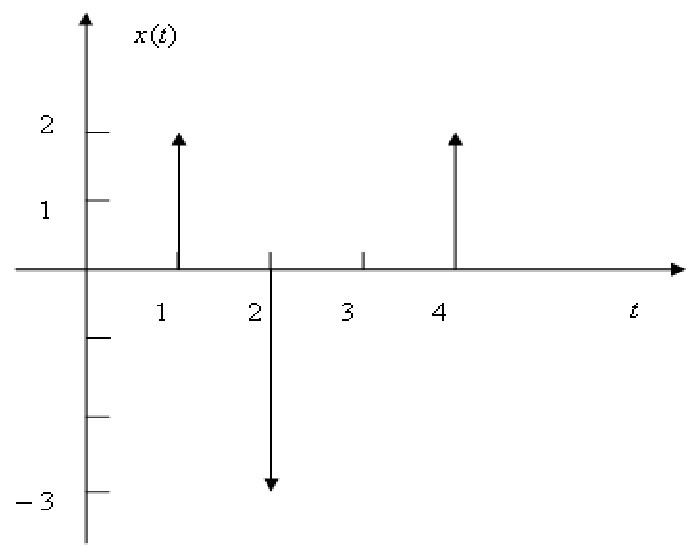
- Scaling Property $$\frac{d(ku(t))}{dt} = k\delta(t)$$

- Example: to derive the 1st derivative of x(t)



$$x(t) = 2u(t-1) - 3u(t-2) + 2u(t-4)$$

$$\frac{dx(t)}{dt} = 2\delta(t-1) - 3\delta(t-2) + 2\delta(t-4)$$



- Example: to determine the following signals/values

1. $(t^2-1)\delta(t-2)$
2. $\int_{-3}^{3}(t^2-1)\delta(t-2)dt$
3. $x[n-3]\delta[n+1]$
4. $\int_{-3}^{t}(\tau^2-1)\delta(\tau-2)d\tau$

Topic

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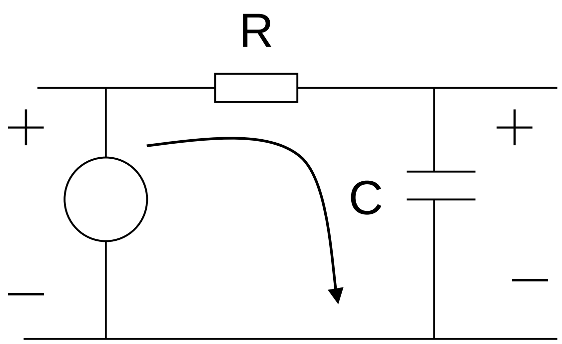
1.5.1 System Modeling

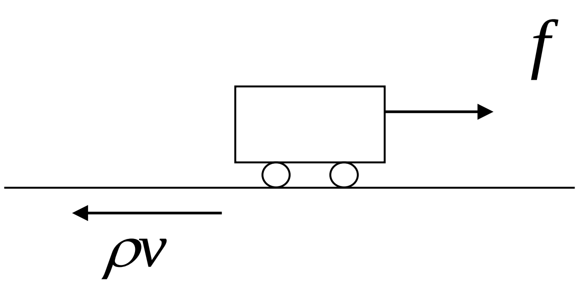
- RLC Circuit



$$\because i(t)=\frac{V_s(t)-V_c(t)}{R} \quad i(t)=C\cdot\frac{dV_c(t)}{dt}$$

$$\therefore \frac{dV_c(t)}{dt}+\frac{1}{RC}V_c(t)=\frac{1}{RC}V_s(t)$$

- Mechanism System



$$\because \frac{dv(t)}{dt}=\frac{1}{m}[f(t)-\rho v(t)]$$

$$\therefore \frac{dv(t)}{dt}+\frac{\rho}{m}v(t)=\frac{f(t)}{m}$$

1.5.1 System Modeling

- Observations:

$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

  - Very different physical systems may be modeled mathematically in very similar ways.
  - Very different physical systems may have very similar mathematical descriptions.

1.5.1 System Modeling

- Typical Systems and their block illustrations
  - Amplifier

    $y(t)=cx(t)$
  - Adder

    $y(t)=x1(t)+x2(t)$
  - Multiplier

    $y(t)=x1(t)*x2(t)$
  - Differentiator/Difference

    $y(t)=dx(t)/dt$, $y[n]=x[n]-x[n-1]$
  - Integrator/Accumulator

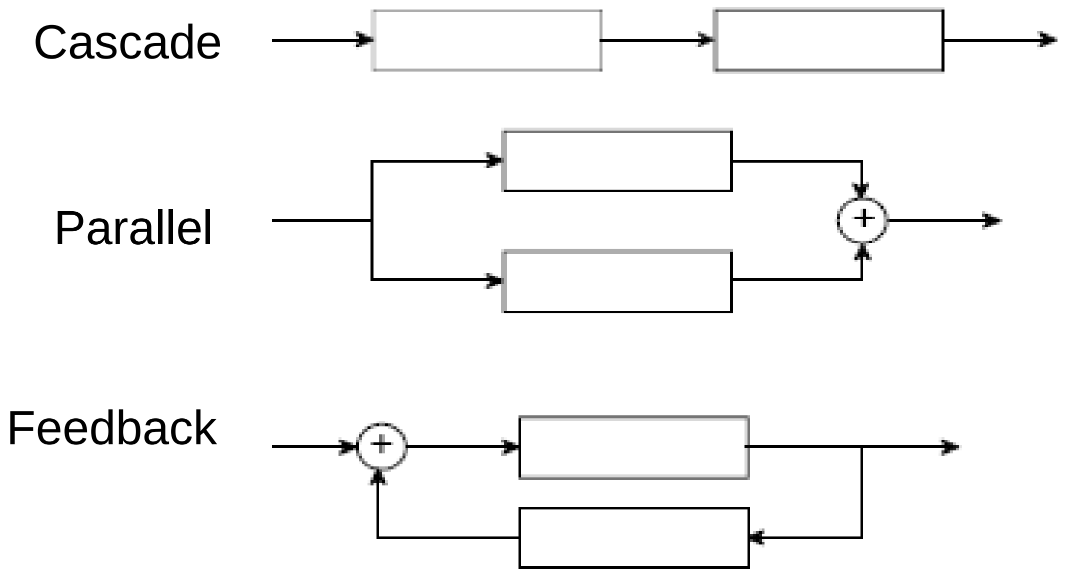
    …

1.5.2 System Analysis

- Memory vs. Memoryless
- Invertibility: Invertible vs. noninvertible
- Causality: Casual vs. non-Casual
- Linearity: Linear vs. non-Linear
- Time-invariance: Time-invariant vs. Time-varying
- Stability: Stable vs. non-Stable

1.5.3 System Interconnections

- The concept of system interconnections
  - To build more complex systems by interconnecting simpler subsystems
  - To modify response of a system
- Signal flow (Block) diagram



Institute of Media,
Information, and Network

Topic

- 1.0 INTRODUCTION
- 1.1 CONTINUOUS-TIME AND DISCRETE-TIME SIGNALS
- 1.2 TRASFORMATION OF INDEPENDENT VARIABLE
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1.6.1 Systems with and without Memory

- Systems with memory: if the current output of the system is dependent on future and/or past values of the inputs and/or outputs, e.g.:
  - Capacitor system:

$$u(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \qquad\qquad y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau$$

  - Accumulator system:

$$y[n] = \sum_{k=-\infty}^{n} x[k] \qquad\qquad y[n] = \sum_{k=-\infty}^{n-1} x[k] + x[n] = y[n-1] + x[n]$$

- Memoryless systems: if the current output of the system is dependent on the input at the same time, e.g.
  - Identity system:

$$y(t) = x(t) \qquad\qquad y[n] = x[n]$$

- Examples: to determine the memory property of the following systems:
  - Amplifier, adder, multiplier
  - Integrator, accumulator, differentiator, time inverse system, time scalar, decimator, interpolator, …

1.6.2 Invertibility: Inverse vs. non-inverse systems

- Inverse systems: distinct inputs lead to distinct outputs, e.g.

$$y(t) = 2x(t) \quad - \quad w(t) = \frac{1}{2} y(t)$$

- Non-inverse systems: distinct inputs may lead to the same outputs, e.g.

$$y(t) = x^2(t) \qquad\qquad y[n] = 0$$

- Importance of the concept: encoding for channel coding or lossless compress

1.6.3 Causality

- A system is causal if the output does not anticipate future values of the input, i.e., if the output at any time depends only on values of the input up to that time
  - All real-time physical systems are causal, because time only moves forward. Effect occurs after cause. (Imagine if you own a non-causal system whose output depends on tomorrow's stock price.)
  - Causality does not apply to spatially varying signals. (We can move both left and right, up and down.)
  - Causality does not apply to systems processing recorded signals, e.g. taped sports games vs. live show.

1.6.3 Causality

- Mathematical definition: A system $x(t) \rightarrow y(t)$ is casual if

  when $x_1(t) \rightarrow y_1(t)$ $x_2(t) \rightarrow y_2(t)$

  and $x_1(t) = x_2(t)$ for all $t \le t_0$

  Then $y_1(t) = y_2(t)$ for all $t \le t_0$

- If two inputs to a casual system are identical up to some point in time t_0, the corresponding outputs are also equal up to the same time.

- Examples: Considering the causality property of the following signals

$$y(t) = x^2(t-1)$$

[redacted]

$$y(t) = x(t+1)$$

[redacted]

$$y[n] = x[-n]$$

[redacted]

$$y[n] = \left(\frac{1}{2}\right)^{n+1} x^3[n-1]$$

[redacted]

1.6.4 Linearity: Linear vs. non-Linear

- Many systems are nonlinear. For example: many circuit elements (e.g., diodes), dynamics of aircraft, econometric models,…
- But why we investigate linear systems?
  - Linear models represent accurate representations of behavior of many systems (e.g., linear resistors, capacitors, other examples given previously,…)
  - Can often linearize models to examine "small signal" perturbations around "operating points"
  - Linear systems are analytically tractable, providing basis for important tools and considerable insight

- Mathematical definition: A system $x(t) \rightarrow y(t)$ is linear if it has the following additivity property and scaling property （可加性和齐次性）

  If $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$

  Additivity property: $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$

  Scaling property: $ax_1(t) \rightarrow ay_1(t)$

- Equivalent sufficient and necessary condition: superposition property:

  If $x1(t) \rightarrow y1(t)$ and $x2(t) \rightarrow y2(t)$

  then $ax1(t) + bx2(t) \rightarrow ay1(t) + by2(t)$

- Examples, considering the linearity and causality properties of the following signals：

  $y[n] = x2[n]$ Nonlinear, Causal

  $y(t) = x(2t)$ Linear, Non-causal

1.6.5 Time-invariance (TI):

- Informal definition: a system is time-invariant (TI) if its behavior does not depend on what time it is.

- Mathematical definition:
  - For a DT system: A system $x[n] \rightarrow y[n]$ is TI if for any input $x[n]$ and any time shift n_0,

    If $x[n] \rightarrow y[n]$

    then $x[n-n_0] \rightarrow y[n-n_0]$
  - Similarly for a CT time-invariant system,

    If $x(t) \rightarrow y(t)$

    then $x(t-t_0) \rightarrow y(t-t_0)$

- Examples:

  Considering the time-variance property of the following systems:

  - 1. $y[n]=nx[n]$ Time-varying system
  - 2. $y(t)=x2(t+1)$ Time-invariant system

Consider the periodic property of the output of a Time-invariant system with the input signal of period T

- Suppose $x(t + T) = x(t)$

  and $x(t) \rightarrow y(t)$

  Then by TI: $x(t + T) \rightarrow y(t + T)$.

- 3. $y(t)=\cos(x(t))$ Time-invariant system

- 4. Amplitude modulator:

  $y(t)=x(t)\cos\omega t$ Time-varying system

Linear Time-Invariant (LTI) Systems

- By exploiting the superposition property and time –invariant property, if we know the response of an LTI system to some inputs, we actually know the response to many inputs

If $\quad x_k[n] \rightarrow y_k[n]$

Then $\quad \sum_k a_k x_k[n] \rightarrow \sum_k a_k y_k[n]$

- If we can find sets of "basic" signals so that
  - a) We can represent rich classes of signals as linear combinations of these building block signals.
  - b) The response of LTI Systems to these basic signals are both simple and insightful.
- So in this course we will study some powerful analysis tools associated with LTI systems

Stability

- If a system satisfies that the input to the system is bounded, i.e. with finite magnitude, the output is also bounded (BIBO)
- Examples:
  when $|x(t)| < M$, determine whether or not the following systems are stable?

$y(t) = t \cdot x(t)$ Unstable

$y(t) = e^{x(t)}$ Stable

- Homework
  - BASIC PROBLEMS WITH ANSWER: 1.10, 1.11, 1.17, 1.18
  - BASIC PROBLEMS: 1.21, 1.22, 1.25, 1.26, 1.27

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Q & A

Many Thanks